DIGITAL SIGNAL PROCESSING

R19

Unit - I

Introduction to DSP

Signal:- Anything that carries some information is called Signal. (OR)

It is a physical quantity that varies with time, space and so on.

For ex:- Voice Signals, Speech Signal, ECG Signal.

There are basically 2 types of Signals.

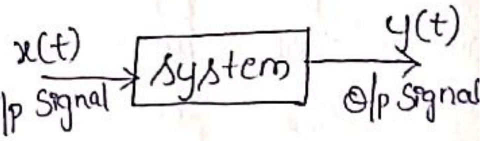
1. continuous time Signals
2. Discrete time Signals

System:- Interconnection of components

It is a physical device that generates response or O/p for a given i/p.

Basically there are 2 types of systems.

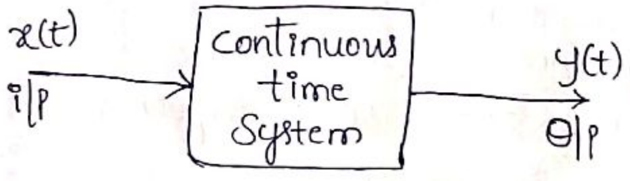
1. continuous time System
2. Discrete time System



Continuous time System

continuous time System is one it can operates in continuous time Signal and produces continuous time Signal as O/p.

Let $x(t)$ & $y(t)$ represents the i/p and O/p of the continuous time System.



$$y(t) = T[x(t)]$$

Discrete time System:- The system is defined as it can operates on discrete time and produces discrete time Signal as the O/p.

Let $x(n)$ and $y(n)$ are the i/p & O/p of the System.

$$y(n) = T[x(n)]$$

Continuous time Signal:- The Signals that are defined for every instant of time are known as continuou time Signals. It can be represented as $x(t)$.

Discrete time Signals:- The Signals that are defined fo discrete instant of time are known as discrete Tim Signals. It can be represented as $x(n)$

These are discrete in Time, continuous in amplitud

Digital Signals:- These Signals are quantized in amplitu and discrete in time are known as digital Signals.

Elementary discrete time Sequences

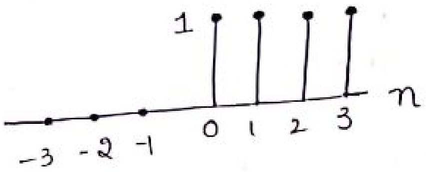
1. unit step Sequence
2. Unit ramp Sequence
3. Unit impulse Sequence
4. Exponential
5. Sinusoidal
6. complex Exponential

Unit step Sequence

The discrete time unit step Sequence can be defined as $u(n) = 1$ for $n \geq 0$

$= 0$ for $n < 0$

The graphical representation of $u(n)$ may be represented as follows.

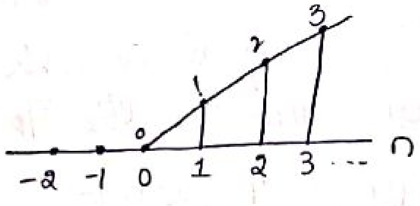


Unit ramp Sequence

The discrete time unit ramp Sequence can be represent as $r(n) = n$ for $n \geq 0$

$= 0$ for $n < 0$

The graphical representation of $r(n)$ may be represented as

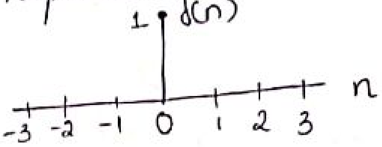


Unit impulse Sequence

The discrete time Unit impulse Sequence can be defined as $\delta(n) = 1$ for $n = 0$

$= 0$ for $n \neq 0$

The graphical representation of $\delta(n)$ can be represented as



Exponential Sequence

The Exponential Signal can be defined as $x(n) = a^n$ for all 'n'. This exponential sequence can be dependend on the value of a i.e., $a > 1$ (or) $0 < a < 1$ (or) $a < 0$

Sinusoidal Sequence

The discrete time Sinusoidal Signal can be defined as $x(n) = A\cos(\omega_0 n + \phi)$. Where ω_0 is frequency measured in rad / samples.

$\phi \rightarrow$ phase (radians)

By using Euler's identify, the above expression can be written as

$$x(n) = \frac{A}{2} e^{j\phi} e^{jn\omega_0} + \frac{A}{2} e^{-j\phi} e^{-jn\omega_0}$$

Complex Exponential Sequence

The complex exponential Sequence can be defined as

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \left[\cos(\omega_0 n + \phi) + j \sin(\omega_0 n + \phi)\right]$$

Representation of discrete time Signals

There are 4 different ways to represent the discrete time Signals.

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

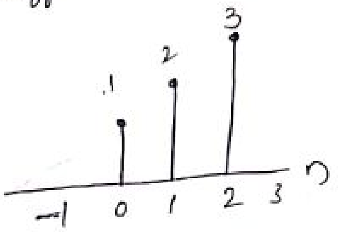
Graphical representation:- Let us consider a discrete time Signal x(n) having different Values i.e.,

$x(-1)=0$

$x(0)=1$

$x(1)=2$

$x(2)=3$



Functional representation:- The discrete time Signal x can be represented in the form of functional representation.

$$x(n)=\begin{cases}-2 & \text{for } n=2\\ -1 & \text{for } n=1\\ 0 & \text{for } n=0\\ 1 & \text{for } n=-1\\ 2 & \text{for } n=-2\end{cases}$$

Tabular representation:- The discrete time Signal x(n) can be represented in tabular form.

n	-1	-2	0	1	2
x(n)	0	1	2	3	4

Sequence representation:- The finite duration Sequence x(n) the time origin for n=0 can be represented by an arrow ↑.

For ex:- $x(n)=\{\underset{\uparrow}{1}, 2, -1, -2\}$

An infinite duration Sequence x(n) the time origi can be represented as $x(n)=\{\ldots\ldots -1, \underset{\uparrow}{0}, 1, 2, 3, \ldots\ldots\}$

Classification of discrete time Signals

1. Energy and power Signals
2. Periodic and Aperiodic Signals
3. Symmetric and Asymmetric Signals
4. Casual and non-Casual Signals

Energy and power Signals

The Energy Signal can be defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

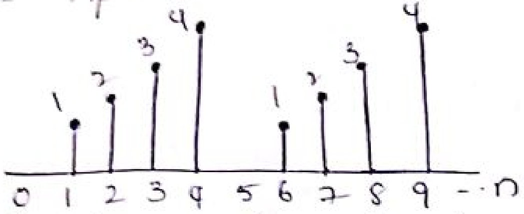
The power Signal can be defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Periodic and Aperiodic Signals

A Signal is Said to be period must Satisfies the condition $x(n) = x(n+N)$. otherwise the Signal is said to be aperiodic.

for N=4



Symmetric and Asymmetric Signals

A Signal is said to be Symmetric Signals or even Symmetry must Satisfies the condition

$$x(n) = x(-n)$$

for ex:- $A \cos \omega_n$

A Signal is Said to be odd Symmetry or Asymmetric Signal must Satisfies the condition is

$$-x(n) = x(-n)$$

(or)

$$x(n) = -x(-n)$$

A discrete time Signal $x(n)$ can be represented as

$$x(n) = x_e(n) + x_o(n)$$

In the above eqⁿ n is replaced by '−n' then

$$x(-n) = x_e(-n) + x_o(-n)$$

According to definition $x(-n) = x_e(n) - x_o(n)$

Combining $x(n)$ & $x(-n)$ then we can get

$$x(n) = x_e(n) + \cancel{x_o(n)}$$
$$x(n) = x_e(n) - \cancel{x_o(n)}$$
$$+$$
$$x(n) + x(-n) = 2x_e(n)$$

$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

Similarly Subtracting $x(n)$ & $x(-n)$ then we can get

$$x(n) = \cancel{x_e(n)} + x_o(n)$$
$$x(-n) = \cancel{x_e(n)} - x_o(n)$$
$$x(n)-x(-n) = 2x_o(n)$$
$$x_o(n) = \frac{x(n)-x(-n)}{2}$$

Casual and non-casual Signals

A Signal is Said to be casual Signal if its Value 'o' for $n<0$. other wise the Signal is Said to be non-casual [for $n \geq 0$].

Ex:- for casual Signal is $x(n)=a^n\,u(n)$

for non-casual Signal $x(n)=a^n\,u(-n+1)$

Classification of discrete time Systems

1) Static and dynamic Systems
2) casual and non-casual Systems
3) Linear and non-linear Systems
4) Time invariant and Time variant Systems
5) FIR and IIR Systems
6) Stable and unstable Systems

Static and dynamic Systems

A discrete Time System is called static if its Outpu at any instant of time depends on the input Sampl at the Same time but not on past and Future Samples of the input, in any other case the System is Said be dynamic System.

The Examples are

$y(n)=a\,x(n)$ for static System

$y(n)=x(n-)+x(n+2)$ for dynamic System.

Casual and non-casual Systems

A System is said to be casual if the output of the system at any time depends only at present and past inputs but does not depend on future inputs. It can be represented as

$$y(n) = f[x(n), x(n-1), x(n-2) \dots]$$

If the output of a system depends on future inputs the system is said to be non-casual. It can be represented as $y(n) = x[2n] + x(n+1)$.

Linear and non-linear Systems

A System that satisfies super position principles is said to be a linear system. The super position principle states that the response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of the outputs of the systems to each of the individual input signal.

A system is linear if and only if

$$T[ax_1(n) + bx_2(n)] = a\,T[x_1(n)] + b\,T[x_2(n)]$$

Here a and b are arbitary constants.

A System that does not satisfy the super position principle is called non-linear system.

Time invariant and Time variant Systems

A System is said to be time invariant or shift invariant if the characteristics of the system does not change with time.

For a time invariant system if $y(n)$ is the response of the system to the input $x(n)$ then response of the system to the input $x(n-k)$ is $y(n-k)$.

$$y(n-k) = T[x(n-k)]$$

$y(n-k) \neq T[x(n-k)]$ then the system is said to be time variant.

A linear time invariant System (LTI) Satisfies bo linearities in the time invariant properties.

FIR and IIR Systems

If the impulse response of the system is of finite duration, then the System is called FIR System.

Ex:- $h(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0 \\ 3 & \text{for } n=1 \end{cases}$

A System has an impulse response for infinite durat is called IIR System.

Ex:- $h(n) = a^n u(n)$

Stable and unstable systems

An LTI System is stable if it produces a boun O/p sequence for every bounded i/p sequence.

If for Some bounded i/p sequence $x(n)$ the O/p is unbounded the System is called unstable. The necessary and sufficient condition for stability is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Problems

1] Test the stability of the system whose impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$.

Sol:- To test the stability the given condition is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Given data

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$= \sum_{n=-\infty}^{\infty} \left|\left[\left(\frac{1}{2}\right)^n u(n)\right]\right|$$

$$= \sum_{n=-\infty}^{0} \left[\left(\frac{1}{2}\right)^n u(n)\right] + \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n u(n)\right]$$

$$= 0 + \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n u(n)\right]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n u(n)$$

$[\because u(n) = 1 \text{ for } n \geq 0]$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

$$= 1 + 1/2 + (1/2)^2 + (1/2)^3 + \cdots\cdots + \infty$$

$$= 1 + x + x^2 + x^3 + \cdots\cdots + \infty = \frac{1}{1-x}$$

$$= \frac{1}{1-1/2} = 2 < \infty$$

∴ The given System is stable.

2] Test the given System for time invariant or not $y(n) = n\,x(n)$.

Sol:- Given data

$$y(n) = n\,x(n)$$

$$y(n-k) \neq T[x(n-k)]$$

∴ The given System is time variant.

3] Determine wheather the following System given by $y(n) = \log_{10}[x(n)]$ is casual or not.

Sol:- Given data

$$y(n) = \log_{10}[x(n)]$$

for $n = 0$

$$y(0) = \log_{10} x(0)$$

for $n = -1$

$$y(-1) = \log_{10} x(-1)$$

∴ The given system is casual system.

4] Deteermine wheather the System defined by $y(n) = x(-n^2-2)$ is time variant or not.

Sol:- Given data

$$y(n) = x(-n^2-2)$$

$$y(n-k) = x\left[-(n-k)^2-2\right]$$

$\therefore$ The given System is time variant.

5] Test the following Systems for time invariant

$$y(n) = n\,x^2(n).$$

Sol:- Given data

$$y(n) = n\ x^2(n)$$

$$y(n-k) \neq T\left[x(n-k)\right]$$

$\therefore$ The given system is time variant.

6] Test wheather the following Signal is periodic or no If periodic find the fundamental period $\sin\sqrt{2}\pi t$.

Sol:- Given $\sin\sqrt{2}\,\pi t$

periodic Signal $\Rightarrow x(t) = x[t+T]$

Let $x(t) = \sin\sqrt{2}\,\pi t = \sin\omega_0 t$

$$\omega_0 = \sqrt{2}\,\pi$$

$$2\pi f = \sqrt{2}\,\pi$$

$$\frac{2\cancel{\pi}}{T} = \sqrt{2}\,\cancel{\pi}$$

$$T = \frac{2}{\sqrt{2}}$$

$$T = \sqrt{2}$$

7] Test wheather the following Signal is periodic or not. If periodic find the fundamental period $\sin 20\pi t + \sin 5\pi t$.

Sol:- Given Data

$$x(t) = \sin 20\pi t + \sin 5\pi t = \sin\omega_1 t + \sin\omega_2 t$$

$$\omega_1 = 20\pi \qquad \omega_2 = 5\pi$$

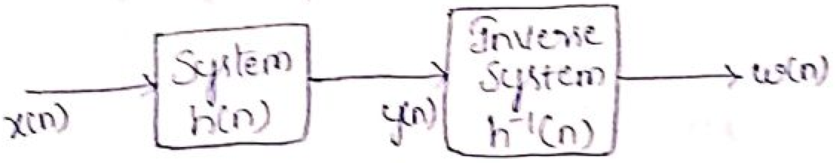
$$\frac{2\cancel{\pi}}{T_1} = 20\cancel{\pi} \qquad \frac{2\cancel{\pi}}{T_2} = 5\cancel{\pi}$$

$$T_1 = \frac{2}{10} = \frac{1}{10} = 0.1 \qquad T_2 = \frac{2}{5} = 0.4$$

$$\therefore T = \frac{T_1}{T_2} = \frac{0.1}{0.4} = 0.25$$

Invertability (or) Inverse System

- A System is Said to be invertable we can generate the i/p Signal x(n) from the o/p of a System y(n).
- It can be generated by using cascading of inverse System with the original System.
- It can be represented as



$$w(n) = y(n) * h^{-1}(n)$$
$$= [x(n) * h(n)] * h^{-1}(n)$$
$$= x(n) * h(n) * h^{-1}(n)$$
$$= x(n)$$

* Determine the convolution of two Sequences

$x(n) = \{3, 2, 1, 2\}$ $\quad h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$

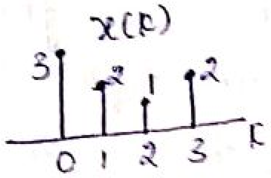
Sol:- Given Sequences

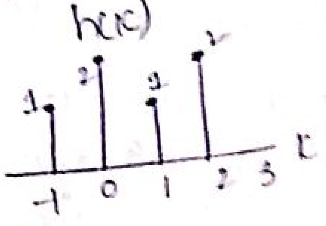
$x(K) = \{3, 2, 1, 2\}$ $\quad h(K) = \{1, \underset{\uparrow\uparrow}{2}, 1, 2\}$

Length of the Sequence $x(K) = 4 = N_1$ $\quad$ Length of the Sequence $h(K) = 4 = N_2$

Total O/P Sequence $= N_1 + N_2 - 1 = 4 + 4 - 1 = 8 - 1 = 7$

$n_1 = 0$ & $n_2 = -1 \Rightarrow n_1 + n_2 = 0 - 1 = -1$



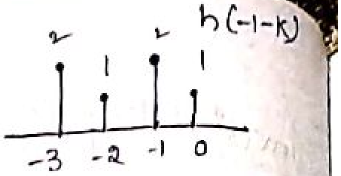


$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$$y(n) = \sum_{K=-1}^{5} x(K)\, h(n-K) \qquad [K = -1, 0, 1, 2, 3, 4, 5]$$

for $n=-1 \Rightarrow y(-1) = \Sigma\, x(k)\, h(-1-k)$

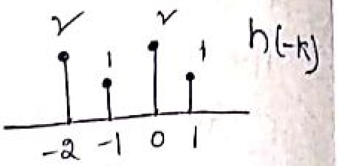
$= 1\times 3 = 3$



$y(0) = \Sigma\, x(k)\, h(-k)$

$= (3\times 2) + (1\times 2)$

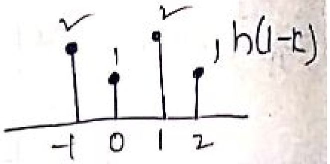
$= 6+2 = 8$



$y(1) = \Sigma\, x(k)\, h(1-k)$

$= (1\times 3) + (2\times 2) + (1\times 1)$

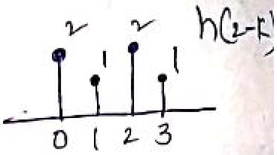
$= 3+4+1 = 8$



$y(2) = \Sigma\, x(k)\, h(2-k)$

$= (3\times 2) + (1\times 2) + (2\times 1) + (1\times 2)$

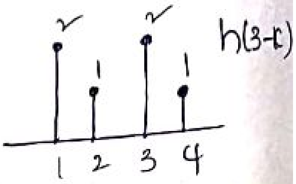
$= 12$



$y(3) = \Sigma\, x(k)\, h(3-k)$

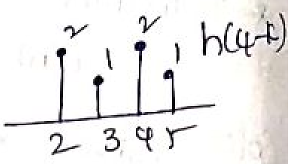
$= (2\times 2) + (1\times 1) + (2\times 2)$

$= 9$



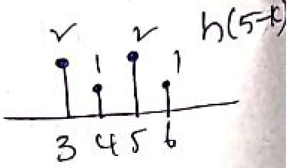
$y(4) = \Sigma\, x(k)\, h(4-k)$

$= 2+2 = 4$



$y(5) = \Sigma\, x(k)\, h(5-k)$

$= 4$



$y(n) = \{3, 8, 8, 12, 9, 4, 4\}$

Convolution

- For a linear time invariant System with the input Sequence x(n) and impulse response h(n) are given we can find the O/P y(n) by using the equation

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$ which is known as Convolution Sum and can be represented as $y(n) = x(n) * h(n)$. Where * represents the convolution operation.

The convolution Sum of two Sequences can be found by using the following Steps.

Step 1:- Choose an initial value of n, the Starting time for Evaluating the O/p Sequence y(n). If x(n) starts at $n=n_1$ and h(n) starts at $n=n_2$ then $n=n_1+n_2$ is considerable choice.

Step 2:- Express both Sequences interms of the index K.

Step 3:- Fold h(K) about K=0 to Obtained h(-K) and shift by 'n' to the right if n is positive and left if n is negative to Obtain h(n-K).

Step 4:- Multiply the two Sequences x(K) and h(n-K) Element by Element and Sum of the products to get y(n).

Step 5:- Increment the index n, Shift the Sequence h(n-K) to right by One Sample and do step 4.

Step 6:- Repeat Step 5 until the Sum of Products is Zero for all the remaining values of 'n'.

Properties of convolution

1. Commutative law

$$x(n) * h(n) = h(n) * x(n)$$

2. Associative law

$$(x(n) * h_1(n)) * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

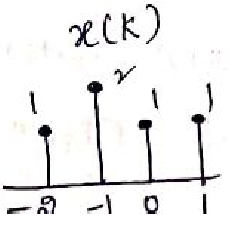
3. Distributive law

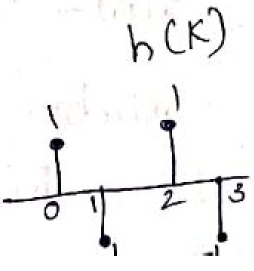
$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

* Find the convolution of two Sequences $x(n)=\{1,2,\underset{\uparrow}{1},1\}$ $h(n)=\{1,-1,1,-1\}$

Sol:- $n_1=-2$ and $n_2=0$

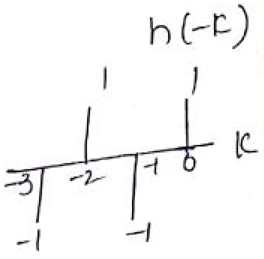
$$n=n_1+n_2=-2+0=-2$$





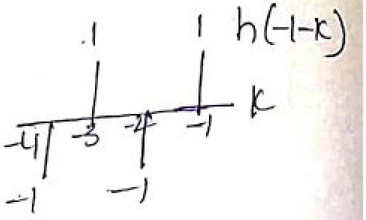
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=-2}^{4} x(k)\, h(n-k)$$



for $\underline{\underline{n=0}}$:- $y(0) = \sum x(k)\, h(-k)$

$= 1-2+1=0$

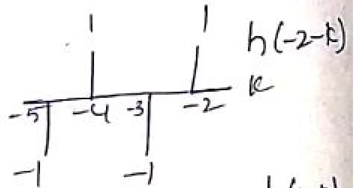


$y(-1) = \sum x(k)\, h(-1-k)$

$= -1+2 = 1$

$y(-2) = \sum x(k)\, h(-2-k)$

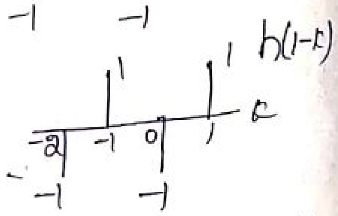
$= 1$



$y(1) = \sum x(k)\, h(1-k)$

$= -1+2-1+1$

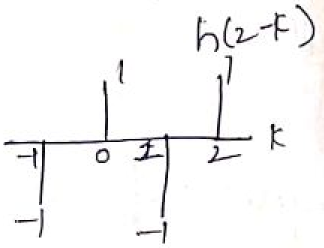
$= 1$



$y(2) = \sum x(k)\, h(2-k)$

$= -2+1-1$

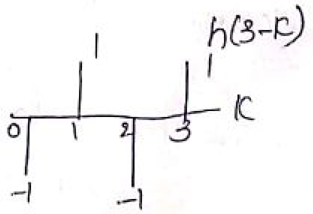
$= -2$



$y(3) = \sum x(k)\, h(3-k)$

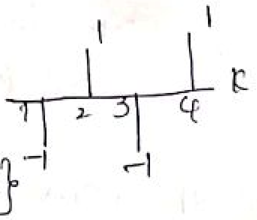
$= -2+1-1$

$= -2$



$y(4) = \sum x(k)\, h(4-k)$

$= -1$



$y(n) = \{1, 1, \underset{\uparrow}{0}, 1, -2, 0, -1\}$

$\underline{\underline{Z}}$ - $\underline{\underline{\text{Transform}}}$

The z-transform of a discrete time signal $x(n)$ is defined as $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$.

Where z is a complex variable in polar form $z = re^{j\omega}$. Where r is the radius of the circle.

The z-value Substitute in the above equation

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left(re^{j\omega}\right)^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, r^{-n} . e^{-j\omega n}$$

- The value of r is consider to be 1 then

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

∴ The z-transform is converted into fourier transform whenever $r=1$.

- This is also called as two sided z-transform.
- The discrete time signal x(n) is considered to be a casual signal then.

$$X_T(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$, this is called One Sided z-transform.

Region of convergence (ROC)

The Set of all values of z for which X(z) gets finite value is called ROC of X(z).

Right hand Sequence:- A Sequence is Said to be right hand Sequence for which $x(n)=0$ for $n < n_0$. Where n_0 may be +ve or -ve but finite. The ROC of right hand Sequence will be Entire z-plane except $z=0$.

For Ex:- $x(n) = \{\underset{\uparrow}{1}, 0, -2, 3, 4\}$

$$X(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$= x(0)\, z^{0} + x(1)\, z^{-1} + x(2)\, z^{-2} + x(3)\, z^{-3} + x(4)\, z^{-4}$$

$$= 1 - 2z^{-2} + 3z^{-3} + 4z^{-4}$$

The ROC of X(z) will be Entire z-plane Except z=0

Left hand Sequence :- A Sequence is Said to be left hand Sequence for which $x(n) = 0$ for $n \geq n_0$. Wher n_0 may be +ve or -ve but finite value.

- The ROC of left hand sequence will be Entire z-plane Except $z = \infty$.

For Ex :- $x(n) = \{-3, -2, -1, \underset{\uparrow}{0}\}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= x(-3) z^{3} + x(-2) z^{2} + x(-1) z^{1} + x(0) z^{0}$$

$$= -3z^{3} - 2z^{2} - z$$

The ROC of $X(z)$ will be Entire z-plane Except $z=\infty$

Two Sided Sequence :- A Signal that has finite duration on both left and right hand Sides is known as two Side Sequence for Such type of Sequence the ROC is Entire z-plane except $z=0$ and $z=\infty$.

For Ex :- $x(n) = \{2, -1, 3, 2, \underset{\uparrow}{1}, 0, 2, 3, -1\}$. Find $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= x(-4) z^{4} + x(-3) z^{3} + x(-2) z^{2} + x(-1) z^{1} + x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4}$$

$$= 2z^{4} - z^{3} + 3z^{2} + 2z + 1 + 2z^{-2} + 3z^{3} - z^{-4}$$

$$X(z) = 2z^{4} - z^{3} + 3z^{2} + 2z + 1 + 2z^{-2} + 3z^{3} - z^{-4}$$

- The ROC of $X(z)$ will be Entire z-plane except $z=0$ and $z=\infty$.

properties of ROC

1] The ROC is a ring/disk in z-plane centered at origin.

2] The ROC cannot contain any poles.

3] The ROC of a casual Sequence will be Entire z-plane except $z=0$.

4] The ROC of a non-casual Sequence will be Entire z-plane Except $z=\infty$.

5] The ROC of finite duration two sided Sequence will be Entire z-plane Except $z=0$ and $z=\infty$.

6] The ROC of in-finite duration two sided sequence will be ring in the z-plane.

7] The ROC of LTI stable System contain unit circle.

8] The ROC must be a connected region.

properties of z-transform

1] Linearity property:-

$$\text{If} \quad Z[x_1(n)] = X_1(z)$$

$$Z[x_2(n)] = X_2(z)$$

$$\text{then} \quad Z[a x_1(n) + b x_2(n)] = a X_1(z) + b X_2(z)$$

proof:- $Z[a x_1(n) + b x_2(n)] = \sum_{n=-\infty}^{\infty} [a x_1(n) + b x_2(n)] z^{-n}$

$$= \sum_{n=-\infty}^{\infty} a x_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} b x_2(n) z^{-n}$$

$$= a \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + b \sum_{n=-\infty}^{\infty} x_2(n) z^{-n}.$$

$$\boxed{Z[a x_1(n) + b x_2(n)] = a X_1(z) + b X_2(z)}$$

2] Time shifting property

$$\text{If} \quad Z[x(n)] = X(z)$$

$$\text{then} \quad Z[x(n-m)] = z^{-m} X(z)$$

Proof :- $Z[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$

Let $n-m = l$

$n = l+m$

$$= \sum_{n=-\infty}^{\infty} x(l)\, z^{-(l+m)}$$

$$= \sum_{n=-\infty}^{\infty} x(l) \cdot z^{-l} \cdot z^{-m}$$

$$= z^{-m} \sum_{n=-\infty}^{\infty} x(l) \cdot z^{-l}$$

$$\boxed{Z[x(n-m)] = z^{-m} X(z)}$$

3] Time Reversal property

If $Z[x(n)] = X(z)$

then $Z[x(-n)] = X(z^{-1})$

proof:- $Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$

Let $-n = l$

$$= \sum_{l=-\infty}^{\infty} x(l)\, z^{l}$$

$$= \sum_{l=-\infty}^{\infty} x(l)\, (z^{-1})^{-l}$$

$$\boxed{Z[x(n)] = X(z^{-1})}$$

4) Differentiation property :- If z-transform of $z[x(n)] = X(z)$ th

$z[n x(n)] = -z \frac{d}{dz} X(z)$

Proof :- $X(z) = \sum_{n=\alpha}^{\alpha} x(n) z^{-n}$

$$\frac{d}{dz} X(z) = \sum_{n=-\alpha}^{\alpha} x(n) - n . z^{-n-1}$$

$$\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} n x(n) z^{-n}\left(\frac{-1}{z}\right)$$

Multiplying $-z$ on both sides of eqn.

$$-z\frac{d}{dz}X(z) = \cancel{-z}\sum_{n=-\infty}^{\infty} n x(n) z^{-n}\left(\frac{-1}{\cancel{z}}\right)$$

$$-z\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} n x(n) z^{-n}$$

$$= z[nx\cdot(n)]$$

$$z[nx(n)] = -z\frac{d}{dz}X(z)$$

5. convulation property :-

If $z[x(n)] = X(z)$ and $z[h(n)] = H(z)$ then

$z[x(n) * h(n)] = X(z)H(z)$

proof:- convulation of two sequences

$$x(n) * h(n) = y(n)$$

$$y(n) = \sum_{k=\infty}^{\infty} x(k)h(n-k)$$

$$z[x(n) * h(n)] = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x(k)h(n-k)z^{-n}.$$

Interchange the order of summation.

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(n-k) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{l=-\infty}^{\infty} h(l) z^{-l} z^{-k}$$

$$= \sum_{k=-\infty}^{\infty} x(k) z^{-k} \sum_{l=-\infty}^{\infty} h(l) z^{-l}$$

$$= X(z)H(z)$$

6. Initial value theorem:-

If $z[x(n)] = X_+(z)$ then

$x(0) = \underset{z\to\infty}{Lt}\ X_+(z)$ where

$X_+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$ in this eqn.

Sub. in $z = +\infty$ all the values are vanish except $x(0)$.

7. Final value theorem :-

If $z[x(n)] = X_+(z)$ then $x(\infty) = \lim_{z \to 1} (z-1) X_+(z)$

System Functions :- In general the system is described by a linear constant co-efficient differential equation of the form

$$\sum_{k=0}^{N} a_k y(n-k)$$

$$= \sum_{k=0}^{m} b_k x(n-k)$$

$$a_0 y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

Let $a_0 = 1$

$$\boxed{y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)}$$

Taking z- transform on b.s & applying time shifting property we can get

$$y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z) \left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k X(z) z^{-k}.$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{N} a_k \cdot z^{-k}}$$

$$\boxed{H(z) = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{N} a_k \cdot z^{-k}}}$$

* Here $H(z)$ is known as transfer function (or) system function.

* $Y(z)$ is the z-transform of the o/p sequence $y(n)$ and $X(z)$ is the z-transform of the i/p sequence $y(n)$.

* By using the system function we can generate poles & zeroes. i.e., the zeros of the system function $H(z)$ are the values of 'z'. for which $H(z)=0$

* The poles of the system function are the values of 'z' for which $H(z)=\infty$.

* The system function contains both poles & zeroes and hence the corresponding system is called pole zero system with 'n' poles & 'm' zeroes.

(P) Find the system function and impulse response of the system describe by the difference eqn $y(n)=1/5\,y(n-1)+x(n)$

Sol) $y(n) = 1/5\,y(n-1)+x(n)$

Applying z-transform on b.s

$$Y(z)=1/5\,Y(z)z^{-1}+X(z)$$

$$Y(z)\left[1-1/5\,z^{-1}\right]=X(z) \Rightarrow \frac{X(z)}{Y(z)}=\frac{1}{1-1/5\,z^{-1}}$$

$$\boxed{H(z)=\frac{1}{1-1/5\,z^{-1}}}$$

Applying inverse z-transform on b.s

$$\boxed{h(n)=(1/5)^n u(n)}$$

(P) Find a z-transform of response $x(n)=(1/3)^{n-1}u(n-1)$

Sol) Given data

$$x(n)=(1/3)^{n-1}u(n-1)$$

$x(n) = (1/3)^n u(n)$

$X(z) = \frac{1}{1-1/3 z^{-1}}$

By using time shifting property

$z\left[(1/3)^{n-1} u(n-1)\right] = z^{-1} X(z)$

$= z^{-1} \frac{z}{z-\frac{1}{3}1/3}$

$$\boxed{= \frac{1}{z-1/3}}$$

Ⓟ Find z- transform of sequence $x(n) = n \cdot a^n u(n)$

Sol Given data,

$x(n) = na^n u(n)$

$z\left[a^n u(n)\right] = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$

By using differenciate property

$z\left[n x(n)\right] = -z \frac{d}{dz} X(z)$

$= -z \frac{d}{dz}\left(\frac{z}{z-a}\right)$

$= -z\left[\frac{z-a-z}{(z-a)^2}\right]$

$$\boxed{z\left[n x(n)\right] = az/(z-a)^2}$$

Time Response Analysis of Discrete time Systems :-

* The difference eqn of nth order discrete time system can be written as

$$\sum_{K=0}^{N} a_K y(n-K) = \sum_{K=0}^{M} b_K x(n-K)$$

* If the co-efficient $a_0 \neq 1$ the solution of the difference eqn. can be apressed as some of two parts given by

$$y(n) = y_h(n) + y_p(n)$$

Where $y_h(n)$ is known as homogeneous solution and $y_p(n)$ is called particular solution.

Natural Response (or) Zero i/p Response :-

* The natural response is the solution of the above difference equation with $x(n)=0$

∴ For a discrete time system the natural response is the solution of homogeneous equation.

i.e.,) $$\sum_{K=0}^{N} a_K y(n-K) = 0$$

* The solution of this equation is of the form $y_h(n) = \lambda^n$

sub. these value in the above eqn. we can get

$$\sum_{K=0}^{N} a_K \lambda^{n-K} = 0$$

$$a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{N-1}\lambda^{n-(N-1)} + a_N\lambda^{n-N} = 0$$

Let, $a_0 = 1$

$$\lambda^{n-N}\left[\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N\right] = 0$$

Which gives,

$$\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N = 0.$$

The above eqn. is characteristic eqn. of the system

∴ The nth order characteristic eqn. can be expressed in characterised form as

$$(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_N)=0$$

* Where, $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_N$ are called roots of the characteristic equation (or) Eigen values of the system.

* The nature of the roots will be real, imaginary and complex roots.

* If the roots $\lambda_1, \lambda_2 \ldots \lambda_N$ are distinct in that case the general Solution of homogeneous eqn.

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n + C_3\lambda_3^n + \ldots + C_N\lambda_n^N$$

Where, $C_1, C_2, C_3 \ldots C_N$ are orbitary constants.

* If the roots are repeated for example if the roots of the characteristic eqn. $\lambda_1=-2$ & $\lambda_2=-2$ and $\lambda_3=2$ then the solution will be

$$y_h(n) = [C_1+C_2 n]\lambda_1^n + C_3\lambda_3^n$$

* If the roots are complex then $\lambda_1 = a+ib$ $\lambda_2 = a-ib$

$$y_h(n) = r[A_1\cos\omega\theta + A_2\sin\omega\theta]$$

$$r = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Where A_1 & A_2 are constants.

***

Ⓟ Find the natural response of the system described by the difference equation $y(n)+2y(n-1)+y(n-2) = x(n)+x(n-1)$ with initial conditions $y(-1) = y(-2) = 1$

Sol Given diff eqn,

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y(n) + 2y(n-1) + y(n-2) = 0$

$\lambda^n + 2\lambda^{n-1} + \lambda^{n-2} = 0$

$\lambda^{n-2}[\lambda^2 + 2\lambda + 1] = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0$

$\lambda_1 = -1, \; \lambda_2 = -1$

$y(n) = [c_1 + c_2 n](-1)^n$

By applying initial conditions

$y(0) = c_1$

$y(1) = (c_1 + c_2)(-1)$

$= -c_1 - c_2$

$y(n) + 2y(n-1) + y(n-2) = 0$

Sub $y(n) = 0$

$y(0) + 2y(0-1) + y(0-2) = 0$

$y(0) + 2 + 1 = 0$

$y(0) = -3$

$y(1) + 2y(0) + y(-1) = 0$

$y(1) - 6 + 1 = 0$

$y(1) = 5$

$\boxed{c_1 = -3}$

$5 = -c_1 - c_2$

$5 = 3 - c_2$

$\boxed{c_2 = -2}$

$\boxed{y_h(n) = [-3 + (-2)n](-1)^n u(n)}$

Ⓟ Find the natural response of the system described by the difference equation $y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$ with initial conditions $y(-1)=y(-2)=1$

Sol Given diff. eqn

$$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$$

$$y(n)-4y(n-1)+4y(n-2)=0$$

$$\lambda^n-4\lambda^{n-1}+4\lambda^{n-2}=0$$

$$\lambda^{n-2}[\lambda^2-4\lambda+4]=0$$

$$\lambda^2-4\lambda+4=0$$

$$(\lambda-2)^2=0$$

$$\lambda_1=2 \; ; \; \lambda_2=2 .$$

$$y(n)=[c_1+c_2 n](\lambda)^n$$

$$y(n)=[c_1+c_2 n]\,(2)^n$$

By applying initial conditions

$$y(0)=[c_1](2)^0=c_1$$

$$y(1)=[c_1+c_2](2)^1$$

$$=[c_1+c_2](2)$$

$$y(n)-4y(n-1)+4y(n-2)=0$$

$$y(0)-4y(-1)+4y(-2)=0$$

$$y(0)-4+4=0$$

$$y(0)=0$$

$$y(1)-4y(0)+4y(-1)=0$$

$$y(1)-0+4(1)=0$$

$$y(1)=-4$$

$$c_1=0$$

$2(c_1+c_2) = -4$

$0+2c_2 = -4$

$$\boxed{c_2 = -2}$$

$$\boxed{y_h(n) = [-3+(-2)^n](-1)^n u(n)}$$

Forced Response :-

The forced Response of the system is oblained by Suming the particular solution and homogenous solution and finding the Coefficients the homogeneous solution So that the Combined response $y_h(n)+y_p(n)$ satisfys the Zero initial Conditions.

General form of particular Solution for Several types of inputs as shown in table.

i/p signal	$y_p(n)$ particular Solution
$\delta(n)$	0
A(step i/p)	K
AM^n	KM^n
An^m	$K_0 n^m + K_1 n^{m-1} + K_2 n^{m-2} + \cdots + KM$
$A^n N^m$	$A^n[K_0 n^m + K_1 n^{m-1} + \cdots + km]$
$A\cos\omega n$ $A\sin\omega n$	$C_1\cos\omega n + C_2\sin\omega n$

Where A, M, N and C_1 & C_2 are Constants.

1) find the forced response of the system described by the difference equation

$y(n)+2y(n-1)+y(n-2) = x(n)+x(n-1)$ for a given i/p

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

Sol given data the difference eqn is

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y_f(n) = y_h(n) + y_p(n)$

$y_h(n) = [c_1 + c_2](-1)^n$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

$y_p(n) = K\left(\frac{1}{2}\right)^n u(n)$

$y_f(n) = [c_1 + c_2](-1)^n + K\left(\frac{1}{2}\right)^n u(n)$

$K\left(\frac{1}{2}\right)^n + 2K\left(\frac{1}{2}\right)^{n-1} + K\left(\frac{1}{2}\right)^{n-2} = \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1}$

$n-2=0$
$n=2$

$K\left(\frac{1}{2}\right)^2 + 2K(1/2) + K = (1/2)^2 + 1/2$

$K(1/4) + K + K = \frac{1}{4} + 1/2$

$K\left(\frac{1}{4} + 2\right) = 3/4$

$K = 1/3$

$y_p(n) = 1/3\left(\frac{1}{2}\right)^n u(n)$

Inorder to determine the values of c_1 & c_2 by applying inital Conditions.

$y_f(n) = [c_1 + c_2 n](-1)^n + K(1/2)^n u(n)$

$y(0) = [c_1](1) + 1/3(1)$

$y(0) = c_1 + 1/3$

$y(1) = (c_1+c_2)(-1) + (1/3)(1/2)$

$y(1) = -c_1 - c_2 + 1/6$

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y(0) + 2y(-1) + y(-2) = x(0) + x(-1)$

$y(0) + 2(0) = 1+0$

$y(0) = 1$

$c_1 + 1/3 = 1$

$c_1 = 1 - 1/3 = 2/3$

$y(1) + 2y(0) + y(-1) = x(1) + x(0)$

$y(1) + 2(1) + 0 = 1/2 + 1$

$y(1) + 2 = 3/2$

$y(1) = -1/2$

$-c_1 - c_2 + 1/6 = -1/2$

$-c_2 = -1/2 - 1/6 + 2/3$

$c_2 = 0$

$$\therefore \boxed{y_f(n) = 2/3 + 1/3(1/2)^n u(n)}$$

2) find the forced response of the system described by the difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the i/p is $x(n) = (-1)^n u(n)$.

Sol given difference equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$y_f(n) = y_n(n) + y_p(n)$

$y_h(n) = [C_1 + C_2 n](2)^n$

$y_p(n) = K(-1)^n u(n)$

$y_f(n) = [C_1 + C_2 n](2)^n + K(-1)^n u(n)$

$K(-1)^n - 4K(-1)^{n-1} + 4K(-1)^{n-2} = (-1)^n - (-1)^{n-1}$

$n-2=0$

$n=2$

$K(-1)^2 - 4K(-1)^1 + 4K(-1)^0 = (-1)^2 - (-1)^1$

$K + 4K + 4K = 1+1$

$9K = 2$

$K = 2/9$

$y_p(n) = \frac{2}{9}(-1)^n u(n)$

Inorder to determine the values of C_1 & C_2 by applying initial Conditions

$y_f(n) = [C_1 + C_2 n](2)^n + K(-1)^n u(n)$

$y(0) = [C_1 + 0](2)^0 + \frac{2}{9}(-1)^0 \cdot 1$

$y(0) = C_1 + \frac{2}{9}$

$y(1) = (C_1 + C_2)(2) + \frac{2}{9}(-1)^1$

$y(1) = (C_1 + C_2)(2) - \frac{2}{9}$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$

$y(0) = 1$

$C_1 + \frac{2}{9} = 1$

$C_1 = 1 - 2/9$

$C_1 = 7/9$

$y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4 + 0 = (-1) - 1$

$y(1) = -2 + 4$

$y(1) = 2$

$2C_1 + 2C_2 - 2/9 = 2$

$2(7/9) + 2C_2 = 2 + 2/9$

$\frac{14}{9} + 2C_2 = 20/9$

$2C_2 = \frac{20}{9} - \frac{14}{9} = 6/9$

$C_2 = 1/3$

$$\boxed{y_f(n) = \left[\frac{7}{9} + \frac{1}{3}n\right](2)^n + \left(\frac{2}{9}\right)(-1)^n u(n)}$$

Frequency Response Analysis of Discrete time Systems:-

The output y(n) of any linear time invariant System to an input signal x(n) Can be obtained by using Convolution Sum $y(n) = \sum_{k=-\alpha}^{\alpha} h(k)\, x(n-k)$.

Where h(n) impulse response of the System. Let us Consider a Complex exponential signal $x(n) = e^{j\omega n}$ as input to the System then the output is given by

$$y(n) = \sum_{k=-\alpha}^{\alpha} h(k)\, e^{j\omega(n-k)}$$

$$= \sum_{k=-\alpha}^{\alpha} h(k)\, e^{j\omega n} \cdot e^{-j\omega k}$$

$$y(n) = e^{j\omega n} \sum_{k=-\alpha}^{\alpha} h(k)\, e^{-j\omega k}$$

$$y(n) = e^{j\omega n} H(e^{j\omega})$$

where $H(e^{j\omega}) = \sum_{k=-\alpha}^{\alpha} h(k)\, e^{-j\omega k}$

The Quantity $H(e^{j\omega})$ is called frequency response of the System. the frequency response is a Complex valued function it can be expressed in polor form as $H(e^{j\omega}) = |H(e^{j\omega})|\, e^{j\theta(\omega)}$.

where $|H(e^{j\omega})|$ is called the magnitude response and it is an even function of 'ω'.

i.e., $|H(e^{j\omega})| = |H(e^{-j\omega})|$

* $\theta(\omega)$ is called phase response of the System and it is an odd function of 'ω'.

i.e., $\theta(\omega) = -\theta(-\omega)$

UNIT - II

Discrete fourier Series

Consider a Periodic Sequence $x_p(n)$ with a Period N.

i.e $x_p(n) = x_p(n+lN)$

where l is an Integer.

The Periodic Sequence Can be represented as weighted sum of Complex exponential whose frequency are Integer multiples of fundamental frequency $2\pi/N$.

$$\therefore e^{j2\pi kn/N} = e^{j2\pi k(n+lN)/N}$$

$\therefore$ $x_p(n)$ can be defined as $\boxed{x_p(n) = 1/N \sum_{k=0}^{N-1} X_p(k) e^{j2\pi kn/N}}$

where $X_p(k)$ are discrete fourier series Co-efficient for

$k = 0$ to $n-1$
$= 0, 1, 2, 3 \ldots n-1$

$$\boxed{X_p(k) = \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi kn/N}}$$

$\therefore$ The above two equation are the discrete fourier Series Pa

It Can be represented as $DFS[x_p(n)] = X_p(k)$

Properties:

1) Linearity Property :-

If $DFS[x_1p(n)] = X_1p(k)$

$DFS[x_2p(n)] = X_2p(k)$

Then $DFS[a x_{1p}(n) + b x_2p(n)] = aX_{1p}(k) + bX_{2p}(k)$

2) Time shifting property

If $DFS[x_p(n)] = X_p(k)$

Then $DFS[x_p(n-m)] = e^{-j2\pi km/N} X_p(k)$

Periodic Convolution Property :-

If $DFS\,(x_{1p}(n)] = X_{1p}(k)$

$DFS\,[x_{2p}(n)] = X_{2p}(k)$

Then $DFS\,[x_{1p}(n) * x_{2p}(n)] = X_{1p}(k)\; X_{2p}(k)$

Symmetry Property :-

If $DFS\,[x_p(n)] = X_p(k)$

$DFS\,[x_p^*(n)] = X_p^*(-k)$

$DFS\,[x_p^*(-n)] = X_p^*(k)$

Then

$$DFS\,[Re\, x_p(n)] = DFS\left[\frac{x_p(n) + x_p^*(n)}{2}\right] = \frac{X_p(k) + X_p^*(-k)}{2} = X_{pe}(k)$$

$$DFS\,[j\,Im\, x_p(n)] = DFS\left[\frac{x_p(n) - x_p^*(n)}{2}\right] = \frac{X_p(k) - X_p^*(-k)}{2} = X_{po}(k)$$

Consider: $x_p(n) = x_{pe}(n) + x_{po}(n)$

then $$x_{pe}(n) = \frac{x_p(n) + x_p^*(-n)}{2}$$

$$x_{po}(n) = \frac{x_p(n) - x_p^*(-n)}{2}$$

$$DFS\,[x_{pe}(n)] = DFS\left[\frac{x_p(n) + x_p^*(-n)}{2}\right] = \frac{X_p(k) + X_p^*(k)}{2} = Re\, X_p(k)$$

$$DFS\,[x_{po}(n)] = DFS\left[\frac{x_p(n) - x_p^*(-n)}{2}\right] = \frac{X_p(k) - X_p^*(k)}{2} = j\,Im\, X_p(k)$$

Discrete fourier Transform (DFT) :-

The discrete fourier transform of sequence $x(n)$ is given by

$$\boxed{\begin{array}{l} x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N} \quad \text{and} \\ X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \end{array}}$$

let us (Consider) define a term $W_N = e^{-j2\pi/N}$ is called twiddle factor.

$$W_N = e^{-j2\pi/N} = \cos 2\pi/N - j\sin 2\pi/N$$

The magnitude of twiddle factor will be

$$|W_N| = |\cos 2\pi/N - j\sin 2\pi/N| \text{ and}$$

Phase $W_N = \angle e^{-j2\pi/N}$

* In terms of twiddle factor the sequence of Discrete fourier transform of x(n) is defined as

$$\boxed{\begin{aligned} x(n) &= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk} \\ X(k) &= \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \end{aligned}}$$

∴ The representation of discrete fourier Transform is

$$DFT[x(n)] = X(k)$$

$$x(n) = IDFT[X(k)]$$

1) Find the DFT of a sequence $x(n) = \{1,1,0,0\}$ and IDFT of the sequence $Y(k) = \{1,0,1,0\}$

Sol Given sequence $x(n) = \{1,1,0,0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

Given N and L ⇒ $N = L = 4$

Consider $X(0) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(0)n/4}$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0$$

$$= 2.$$

$$x(1) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(1)n/\cancel{4}2}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$$

$$= x(0)\, e^{-j\pi(0)/2} + x(1)\, e^{-j\pi(1)/2} + x(2)\, e^{-j\pi(2)/2} + x(3)\, e^{-j\pi(3)/2}$$

$$= 1.1 + 1.e^{-j\pi/2} + 0 + 0$$

$$= 1+1\,[\cos\pi/2 - j\sin\pi/2)$$

$$= 1+[0-j.1]$$

$$= 1-j.$$

$$x(2) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(2)n/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)\, e^{-j\pi(0)} + x(1)\, e^{-j\pi(1)} + x(2)\, e^{-j\pi(2)} + x(3)\, e^{-j\pi(3)}$$

$$= 1.1 + 1.e^{-j\pi} + 0 + 0$$

$$= 1+(\cos\pi - j\sin\pi)$$

$$= 1+[-1-j(0)]$$

$$= 1-1 = 0$$

$$x(3) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(3)n/\cancel{4}2}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j3\pi n/2}$$

$$= x(0)\, e^{-j3\pi(0)/2} + x(1)\, e^{-j3\pi(1)/2} + x(2)\, e^{-j3\pi(2)/2} + x(3)\, e^{-j3\pi(3)/2}$$

$$= 1+1.e^{-j3\pi/2}$$

$$= 1+1.(\cos 3\pi/2 - j\sin 3\pi/2)$$

$$= 1+[0-j(-1))$$

$$= 1+j.$$

$$X(k) = \{2,\ 1-j,\ 0,\ 1+j\}$$

* Given Sequence $Y(k) = \{1,0,1,0\}$

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k)\, e^{j2\pi kn/N}$$

$$y(0) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j2\pi(0)n/4}$$

$$= \frac{1}{4} \sum_{k=0}^{3} Y(k)\cdot 1$$

$$= \frac{1}{4}[Y(0) + Y(1) + Y(2) + Y(3)]$$

$$= \frac{1}{4}[1+0+1+0)$$

$$= \frac{1}{4}(2) = \underline{\underline{\frac{1}{2}}}$$

$$Y(1) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j2\pi(1)n/4}$$

$$= \frac{1}{4}\left[Y(0)\, e^{j2\pi(0)/4} + Y(1)\, e^{j2\pi/4} + Y(2)\, e^{j\cancel{2}\pi(\cancel{2})/\cancel{4}} + Y(3)\, e^{j2\pi(3)/4}\right]$$

$$= \frac{1}{4}[1\cdot 1 + 0 + 1\cdot e^{j\pi} + 0)$$

$$= \frac{1}{4}[1 + \cos\pi + j\sin\pi] = \frac{1}{4}[1 + (-1+0)] = 0$$

$$Y(2) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j\cancel{2}\pi(\cancel{2})n/\cancel{4}}$$

$$= \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{jn\pi}$$

$$= \frac{1}{4}\left[Y(0)\, e^{j(0)\pi} + Y(1)\, e^{j(1)\pi} + Y(2)\, e^{j(2)\pi} + Y(3)\, e^{j(3)\pi}\right]$$

$$= \frac{1}{4}[1\cdot 1 + 0 + 1\cdot e^{j2\pi} + 0]$$

$$= \frac{1}{4}[1 + (1 + j(0))$$

$$= \frac{1}{4}(2) = \frac{1}{2}$$

$$Y(3) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j\cancel{2}\pi(3)n/\cancel{4}2}$$

$$= \frac{1}{4}\left[\sum_{k=0}^{3} Y(k)\, e^{j3\pi n/2}\right]$$

$$= \frac{1}{4}\left[Y(0)\, e^{j3\pi(0)/2} + Y(1)\, e^{j3\pi(1)/2} + Y(2)\, e^{j3\pi(\cancel{2})/\cancel{2}} + Y(3)\, e^{j3\pi(3)/2}\right]$$

$= \frac{1}{4}[1\cdot1 + 0 + 1\cdot e^{j3\pi} + 0]$

$= \frac{1}{4}[1 + (\cos 3\pi + j\sin 3\pi)]$

$= \frac{1}{4}[1 + (-1 + j(0))$

$y(3) = \frac{1}{4}(0] = 0$

$y(k) = \{\dot{1/2}, 0, 1/2, 0\}$

2) Find 8 point DFT of the Sequence $x(n) = \{1,1,1,1,1,1,0,0\}$

Sol given sequence is $x(n) = \{1,1,1,1,1,1,0,0\}$

$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

$N = 8$

$N-1 = 8-1 = 7$

$X(0) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi(0)n/8}$

$= \sum_{n=0}^{7} x(n)\cdot 1$

$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$

$= 1+1+1+1+1+1+0+0$

$= 6$

$X(1) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi(1)n/\cancel{8}4}$

$= \sum_{n=0}^{7} x(n)\, e^{-j\pi n/4}$

$X(1) = x(0)e^{-j\pi(0)/4} + x(2)e^{-j\pi(1)/4} + x(2)e^{-j\pi(2)/\cancel{4}2} + x(3)e^{-j\pi(3)/4}$
$+ x(4)e^{-j\pi(4)/4} + x(5)e^{-j\pi(5)/4} + x(6)e^{-j\pi(6)/4}$
$+ x(7)\, e^{-j\pi(7)/4}$

$X(1) = 1 + 1\cdot e^{-j\pi/4} + 1\cdot e^{-j\pi/2} + 1\cdot e^{-j3\pi/4} + 1\cdot e^{-j\pi} + 0 + 0$
$+ 1\cdot e^{-j5\pi/4}$

$= 1 + 1[\cos\pi/4 - j\sin\pi/4] + [\cos\pi/2 - j\sin\pi/2] +$
$[\cos 3\pi/4 - j\sin 3\pi/4] + [\cos\pi - j\sin\pi] + [\cos 5\pi/4 - \sin 5\pi/4$

$X(1) = 1 + 1/\sqrt{2} - j\,1/\sqrt{2} + 0 - j - 1/\sqrt{2} - j/\sqrt{2} - 1 - 0 - 1/\sqrt{2} + j/\sqrt{2}$

$= \cancel{1} + \cancel{1/\sqrt{2}} - j/\sqrt{2} - j - 1/\sqrt{2} - \cancel{j/\sqrt{2}} - \cancel{1} - \cancel{1/\sqrt{2}} + \cancel{j/\sqrt{2}}$

$= -1/\sqrt{2} - j/\sqrt{2} - j$

$= -1/\sqrt{2} - j(1 + 1/\sqrt{2})$

$X(1) = -0.707 - j(1.707)$

$$X(2) = \sum_{n=0}^{7} x(n) e^{-j2\pi(2)n/\cancel{8}2}$$

$$= \sum_{n=0}^{7} x(n) e^{-jn\pi/2}$$

$$= x(0) e^{-j\pi(0)/2} + x(1) e^{-j\pi/2} + x(2) e^{-j\pi(\cancel{2})/\cancel{2}} + x(3) e^{-j\pi(3)/2} + x(4) e^{-j\pi(\cancel{4})^{2}/\cancel{2}} + x(5) e^{-j\pi(5)/2} + x(6) e^{-j\pi(6)/2} + x(7) e^{-j\pi(7)/2}$$

$$= 1 + e^{-j\pi/2} + 1 \cdot e^{-j\pi} + e^{-j3\pi/2} + e^{-j2\pi} + e^{-j5\pi/2} + 0 + 0$$

$$= 1 + [\cos\pi/2 - j\sin\pi/2] + [\cos\pi - j\sin\pi] + [\cos 3\pi/2 - j\sin 3\pi/2] + [\cos 2\pi - j\sin 2\pi] + [\cos 5\pi/2 - j\sin 5\pi/2]$$

$= 1 + 0 - j - \cancel{1} - 0 + 0 + \cancel{j} + \cancel{1} - 0 + 0 - \cancel{j}$

$= 1 - j$

$$X(3) = \sum_{n=0}^{7} x(n) e^{-j2\pi(3)n/\cancel{8}4}$$

$$= \sum_{n=0}^{7} x(n) e^{-j3\pi(n)/4}$$

$$= x(0) e^{-j3\pi(0)/4} + x(1) e^{-j3\pi/4} + x(2) e^{-j3\pi(\cancel{2})/\cancel{4}2} + x(3) e^{-j3\pi(3)/4} + x(4) e^{-j3\pi(\cancel{4})/\cancel{4}} + x(5) e^{-j3\pi(5)/4} + x(6) e^{-j3\pi(6)/4} + x(7) e^{-j3\pi(7)/4}$$

$$x(3) = 1 + e^{-j3\pi/4} + e^{-j3\pi/2} + e^{-j9\pi/4} + e^{-j3\pi} + e^{-j5\pi/4} + 0 + 0$$

$$= 1 + [\cos 3\pi/4 - j\sin 3\pi/4] + [\cos 3\pi/2 - j\sin 3\pi/2] +$$

$$[\cos 9\pi/4 - j\sin 9\pi/4] + [\cos 3\pi - j\sin 3\pi] +$$

$$[\cos 15\pi/4 - j\sin 15\pi/4]$$

$$x(3) = 1 + [-1/\sqrt{2} - j(1/\sqrt{2})] + [0 - j(-1)] + [1/\sqrt{2} - j(1/\sqrt{2})]$$

$$+ (-1 - j(0)) + (1/\sqrt{2} - j(-1/\sqrt{2}))$$

$$= \cancel{1} - \cancel{1/\sqrt{2}} - \cancel{j/\sqrt{2}} + 0 + j + 1/\sqrt{2} - j/\sqrt{2} - \cancel{1} - 0 + \cancel{1/\sqrt{2}} + \cancel{j/\sqrt{2}}$$

$$= 1/\sqrt{2} - j/\sqrt{2} + j$$

$$= 1/\sqrt{2} + j(1 - 1/\sqrt{2})$$

$$= 0.707 + 0.293j$$

$$x(4) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi(4)n/8}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\pi}$$

$$= x(0)e^{-j(0)\pi} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi} + x(4)e^{-j4\pi}$$

$$+ x(5)e^{-j5\pi} + x(6)e^{-j6\pi} + x(7)e^{-j7\pi}$$

$$= 1 + [\cos\pi - j\sin\pi] + [\cos 2\pi - j\sin 2\pi] + [\cos 3\pi - j\sin 3\pi]$$

$$+ [\cos 4\pi - j\sin 4\pi] + [\cos 5\pi - j\sin 5\pi] + 0 + 0$$

$$= 1 + [-1 - j(0)] + [1 - j(0)] + [-1 - j(0)] + [1 - j(0)]$$

$$+ [(-1) - j(0)]$$

$$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1}$$

$$= 0$$

$$X(5) = \sum_{n=0}^{7} x(n)\, e^{-j\cancel{2}\pi(5)\,n/\cancel{8}\,4}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j5\pi n/4}$$

$$= x(0)e^{-j5\pi(0)/4} + x(1)e^{-j5\pi(1)/4} + x(2)e^{-j5\pi(\cancel{2})/\cancel{4}2} +$$
$$x(3)e^{-j5\pi(3)/4} + x(4)e^{-j5\pi(9)/4} + x(5)e^{-j5\pi(5)/4} +$$
$$x(6)e^{j5\pi(6)/4} + x(7)e^{-j5\pi(7)/4}$$

$$= 1 + e^{-j5\pi/4} + e^{-j5\pi/2} + e^{-j15\pi/4} + e^{-j5\pi} + e^{-j25\pi/4} + 0 + 0$$

$$= 1 + [\cos 5\pi/4 - j\sin 5\pi/4] + [\cos 5\pi/2 - j\sin 5\pi/2] +$$
$$[\cos 15\pi/4 - j\sin 15\pi/4] + [\cos 5\pi - j\sin 5\pi] +$$
$$[\cos 25\pi/4 - j\sin 25\pi/4]$$

$$= 1 + [-1/\sqrt{2} - j(-1/\sqrt{2})] + [0 - j] + [1/\sqrt{2} + j/\sqrt{2}] + [-1 - j(0)]$$
$$+ [1/\sqrt{2} - j(1/\sqrt{2})]$$

$$= \cancel{1} - \cancel{1/\sqrt{2}} + j/\sqrt{2} - j + \cancel{1/\sqrt{2}} + \cancel{j/\sqrt{2}} - \cancel{1} + 1/\sqrt{2} - \cancel{j/\sqrt{2}}$$

$$= j/\sqrt{2} - j + 1/\sqrt{2}$$

$$= 0.707 - j(1 - 1/\sqrt{2})$$

$$X(5) = 0.707 - j(0.293)$$

$$X(6) = \sum_{n=0}^{7} x(n)\, e^{-j\cancel{2}\pi(\cancel{6}\,3)\,n/\cancel{8}\,\cancel{4}\,2}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j3\pi n/2}$$

$$= x(0)e^{-j3\pi(0)/2} + x(1)e^{-j3\pi(1)/2} + x(2)e^{-j3\pi(\cancel{2})/\cancel{2}} +$$

$x(3)e^{-j3\pi(3)/2} + x(4)e^{-j3\pi(\overset{2}{\cancel{4}})/\cancel{2}} + x(5)e^{-j3\pi(5)/2} +$

$x(6)e^{-j3\pi(\overset{3}{\cancel{6}})/\cancel{2}} + x(7)e^{-j3\pi(7)/2}$.

$X(6) = 1 + e^{-j3\pi/2} + 1e^{-j3\pi} + e^{-j9\pi/2} + e^{-j6\pi} + e^{-j15\pi/2} + 0 + 0$

$= 1 + [\cos 3\pi/2 - j\sin 3\pi/2] + [\cos 3\pi - j\sin 3\pi] +$

$[\cos 9\pi/2 - j\sin 9\pi/2] + [\cos 6\pi - j\sin 6\pi] +$

$[\cos 15\pi/2 - j\sin 15\pi/2]$

$= 1 + (0 - j(-1)) + (-1 - j(0)) + (0 - j(1)) + (1 - j(0)) +$
$(0 - j(-1))$

$X(6) = \cancel{1} + \cancel{j} - \cancel{1} - \cancel{j} + 1 + j$

$= 1 + j$

$X(7) = \sum_{n=0}^{7} x(n)e^{-j\cancel{2}\pi(7)n/\cancel{8}4}$

$= \sum_{n=0}^{7} x(n)e^{-j7\pi n/4}$

$= x(0)e^{-j7\pi(0)/4} + x(1)e^{-j7\pi/4} + x(2)e^{-j7\pi(\cancel{2})/\cancel{4}2} +$

$x(3)e^{-j7\pi(3)/4} + x(4)e^{-j7\pi(\cancel{4})/\cancel{4}} + x(5)e^{-j7\pi(5)/4} +$

$x(6)e^{-j7\pi(6)/4} + x(7)e^{-j7\pi(7)/4}$

$= 1 + e^{-j7\pi/4} + e^{-j7\pi/2} + e^{-j21\pi/4} + e^{-j7\pi} + e^{-j35\pi/4} + 0 + 0$

$= 1 + [\cos 7\pi/4 - j\sin 7\pi/4] + [\cos 7\pi/2 - j\sin 7\pi/2] +$

$[\cos 21\pi/4 - j\sin 21\pi/4] + [\cos 7\pi - j\sin 7\pi] +$

$[\cos 35\pi/4 - j\sin 35\pi/4]$

$= 1 + [1/\sqrt{2} - j(-1/\sqrt{2})] + [0 - j(-1)] + [-1/\sqrt{2} - j(-1/\sqrt{2})] +$

$(-1-j(0)) + (-1/\sqrt{2} - j(1/\sqrt{2}))]$

$= \cancel{1} + \cancel{1/\sqrt{2}} + \cancel{j/\sqrt{2}} + j - 1/\sqrt{2} + j/\sqrt{2} \cancel{-1} - \cancel{1/\sqrt{2}} - \cancel{j/\sqrt{2}}$

$- j - 1/\sqrt{2} + j/\sqrt{2}$

$x(7) = -0.707 + j(1.707)$

$x(k) = \{6, -0.707 - j1.707, 1-j, 0.707 + 0.293j, 0,$
$0.707 - 0.293j, 1+j, -0.707 + j1.707\}$

3) Find IDFT of Sequence $x(k) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0$

Sol given Sequence $x(k) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, e^{j2\pi kn/N}$$

$N = 8$

$N - 1 = 8 - 1 = 7$

$$x(0) = \frac{1}{8} \sum_{k=0}^{7} x(k)\, e^{j2\pi(0)k/8}$$

$$= \frac{1}{8} \sum_{k=0}^{7} x(k).1$$

$x(0) = \frac{1}{8}[x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)]$

$= \frac{1}{8}[5 + 0 + 1 \cancel{-j} + 0 + 1 + 0 + 1 \cancel{+j} + 0]$

$= \frac{1}{\cancel{8}}[\cancel{8}]$

$= 1$

$$x(1) = \frac{1}{8} \sum_{k=0}^{7} x(k)\, e^{j\cancel{2}\pi k(1)/\cancel{8}4}$$

$$= \frac{1}{8} \sum_{k=0}^{7} x(k)\, e^{j\pi k/4}$$

$= \frac{1}{8}\Big[x(0)e^{j\pi(0)/4} + x(1)e^{j\pi(1)/4} + x(2)e^{j\pi(\cancel{2})/\cancel{4}2} + x(3)e^{j\pi(3)/4}$
$+ x(4)e^{j\pi(\cancel{4})/\cancel{4}} + x(5)e^{j\pi(5)/4} + x(6)e^{j\pi/(\cancel{6}^{3})/\cancel{4}^{2}} + x(7)e^{j\pi(7/4)}\Big]$

$$x(1)=\frac{1}{8}\left[5+0+(1-j)e^{j\pi/2}+0\times e^{j3\pi/4}+1\,e^{j\pi}+0\cdot e^{j5\pi/4}+\right.$$
$$\left.(1+j)e^{j3\pi/2}+0\cdot e^{j7\pi/4}\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos\pi/2+j\sin\pi/2\right]+\left[\cos\pi+j\sin\pi\right]+(1+j)\right.$$
$$\left.\left[\cos 3\pi/2+j\sin 3\pi/2\right]\right]$$

$$=\frac{1}{8}\left[5+(1-j)(0+j)+(-1+j(0))+(1+j)(0+j(-1))\right]$$

$$=\frac{1}{8}\left[5+j-j^2-1+(1+j)(-j)\right]$$

$$=\frac{1}{8}\left[5+\cancel{j}-j^2-1-j^2-\cancel{j}\right]$$

$$=\frac{1}{8}\left[5-1+1+1\right]$$

$$=6/8=3/4=0.75$$

$$x(2)=\frac{1}{8}\sum_{K=0}^{7}X(K)e^{j\cancel{2}\pi K(\cancel{2})/\cancel{8}2}$$

$$=\frac{1}{8}\sum_{K=0}^{7}X(K)e^{j\pi K/2}$$

$$x(2)=\frac{1}{8}\left[X(0)e^{j\pi(0)/2}+X(1)e^{j\pi(1)/2}+X(2)e^{j\pi(2)/2}+X(3)e^{j\pi(3)/2}\right.$$
$$\left.+X(4)e^{j\pi(4)/2}+X(5)e^{j5\pi/2}+X(6)e^{j\pi(6)/2}+X(7)e^{j7\pi/2}\right]$$

$$=\frac{1}{8}\left[5+0+(1-j)e^{j\pi}+0+e^{j2\pi}+0+(1+j)e^{j3\pi}+0\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos\pi+j\sin\pi\right]+(\cos 2\pi+j\sin 2\pi)+(1+j)\right.$$
$$\left.(\cos 3\pi+j\sin 3\pi)\right]$$

$$=\frac{1}{8}\left[5+(1-j)(-1+j(0))+(1+j(0))+(1+j)(-1+j(0))\right]$$

$$=\frac{1}{8}\left[5-1+j+1+(1+j)(-1)\right]$$

$$=\frac{1}{8}\left[4+\cancel{j}+1-\cancel{1}-\cancel{j}\right]$$

$$=4/8=1/2=0.5$$

$$x(3)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\cancel{2}\pi k(3)/\cancel{8}4}$$

$$=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j3\pi k/4}$$

$$=\frac{1}{8}\Big[X(0)e^{j3\pi(0)/4}+X(1)e^{j3\pi(1)/4}+X(2)e^{j3\pi(\cancel{2})/\cancel{4}2}+X(3)e^{j3\pi(3)/4}$$
$$+X(4)e^{j3\pi(\cancel{4})/\cancel{4}}+X(5)e^{j3\pi(5)/4}+X(6)e^{j3\pi(\overset{3}{\cancel{6}})/\cancel{4}2}+$$
$$X(7)e^{j3\pi(7)/4}\Big]$$

$$x(3)=\frac{1}{8}\left[5+0+(1-j)e^{j3\pi/2}+0+e^{j3\pi}+0+(1+j)e^{j9\pi/2}+0\right]$$

$$=\frac{1}{8}\Big[5+(1-j)\left[\cos 3\pi/2+j\sin 3\pi/2\right]+\left(\cos 3\pi+j\sin 3\pi\right)$$
$$+(1+j)\left(\cos 9\pi/2+j\sin 9\pi/2\right)\Big]$$

$$=\frac{1}{8}\left[5+(1-j)(0+j(-1))+((-1)+j(0))+(1+j)(0+j)\right]$$

$$=\frac{1}{8}\left[5+(1-j)(-j)+(-1)+(1+j)(j)\right]$$

$$=\frac{1}{8}\left[5-\cancel{j}+j^2-1+\cancel{j}+j^2\right]$$

$$=\frac{1}{8}\left[4-1-1\right]=1/8(2)=1/4=0.25$$

$$x(4)=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j2\pi\cdot k(4)/\cancel{8}}$$

$$=\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\pi k}$$

$$=\frac{1}{8}\Big[X(0)e^{j\pi(0)}+X(1)e^{j\pi(1)}+X(2)e^{j\pi(2)}+X(3)e^{j\pi(3)}+$$

$x(4)e^{j4\pi} + x(5)e^{j5\pi} + x(6)e^{j6\pi} + x(7)e^{j7\pi}]$

$$x(4) = \frac{1}{8}\left[5+0+(1-j)(\cos 2\pi + j\sin 2\pi) + 0 + [\cos 4\pi + j\sin 4\pi) + (1+j)(\cos 6\pi + j\sin 6\pi)\right]$$

$$= \frac{1}{8}\left[5+(1+j)(1+j(0)\} + (1+j(0)) + (1+j)(1+j(0))\right]$$

$$= \frac{1}{8}\left[5+\cancel{1}-\cancel{j}+\cancel{1}+1+\cancel{j}\right]$$

$$= 8(1/8) = 1$$

$$x(5) = \frac{1}{8}\sum_{k=0}^{7} X(k)e^{j2\pi k(5)/\cancel{8}4}$$

$$= 1/8 \sum_{k=0}^{7} X(k)e^{j5\pi k/4}$$

$$x(5) = \frac{1}{8}\left[X(0)e^{j5\pi(0)/4} + X(1)e^{j5\pi(1)/4} + X(2)e^{j5\pi(2)/\cancel{4}2} + X(3)e^{j5\pi(3)/4} + X(4)e^{j5\pi(\cancel{4})/\cancel{4}} + X(5)e^{j5\pi(5)/4} + X(6)e^{j5\pi(\overset{3}{\cancel{6}})/\cancel{4}2} + X(7)e^{j5\pi(7)/4}\right]$$

$$= \frac{1}{8}\left[5+0+(1-j)e^{j5\pi/2} + 0 + e^{j5\pi} + 0 + (1+j)e^{j15\pi/2} + 0\right]$$

$$= \frac{1}{8}\left[5+(1-j)[\cos 5\pi/2 + j\sin 5\pi/2] + (\cos 5\pi + j\sin 5\pi) + (1+j)(\cos 15\pi/2 + j\sin 15\pi/2)\right]$$

$$= \frac{1}{8}\left(5+(1-j)(0+j(1)) + (-1+j(0)) + (1+j)(0+j(-1))\right)$$

$$= \frac{1}{8}\left[5+\cancel{j}-j^2-1-\cancel{j}-j^2\right]$$

$$= \frac{1}{8}\left[4-2j^2\right] = \frac{1}{8}\left[4+2\right] = 6/8$$

$$x(5) = 0.75$$

$$x(6)=\frac{1}{8}\sum_{k=0}^{7}X(k)\,e^{j\cancel{2}\pi k(\cancel{6}^{3})/\cancel{8}\cancel{4}2}$$

$$=\frac{1}{8}\sum_{k=0}^{7}X(k)\,e^{j3\pi k/2}$$

$$=\frac{1}{8}\Big[X(0)e^{j3\pi(0)/2}+X(1)e^{j3\pi/2}+X(2)e^{j3\pi(\cancel{2})/\cancel{2}}+X(3)e^{j3\pi(3)/2}+X(4)e^{j3\pi(\cancel{4})/\cancel{2}}+X(5)e^{j3\pi(5)/2}+X(6)e^{j3\pi(\cancel{6}^{3})/\cancel{2}}+X(7)e^{j3\pi(7)/2}\Big]$$

$$=\frac{1}{8}\Big[5+0+(1-j)\,e^{j3\pi}+0+e^{j6\pi}+0+(1+j)\,e^{j9\pi}+0\Big]$$

$$=\frac{1}{8}\Big\{5+(1-j)[\cos 3\pi+j\sin 3\pi]+[\cos 6\pi+j\sin 6\pi]+(1+j)[\cos 9\pi+j\sin 9\pi]\Big]$$

$$=\frac{1}{8}\Big[5+(1-j)(-1+j(0))+(1+j(0))+(1+j)(-1+j(0))\Big]$$

$$=\frac{1}{8}\Big[5-1+\cancel{j}+\cancel{1}-\cancel{j}-\cancel{1}\Big]$$

$$=4/8=1/2=0.5$$

$$x(7)=\frac{1}{8}\sum_{k=0}^{7}X(k)\,e^{j\cancel{2}\pi k(7)/\cancel{8}4}$$

$$=\frac{1}{8}\sum_{k=0}^{7}X(k)\,e^{j7\pi k/4}$$

$$=\frac{1}{8}\Big[X(0)e^{j7\pi(0)/4}+X(1)e^{j7\pi(1)/4}+X(2)e^{j7\pi(\cancel{2})/\cancel{4}2}+X(3)e^{j7\pi(3)/4}+X(4)e^{j7\pi(\cancel{4})/\cancel{4}}+X(5)e^{j7\pi(5)/4}+X(6)e^{j7\pi(\cancel{6}^{3})/\cancel{4}^{2}}+X(7)e^{j7\pi(7)/4}\Big]$$

$$=\frac{1}{8}\Big[5+0+(1-j)\,e^{j7\pi/2}+0+e^{j7\pi}+(1+j)\,e^{j21\pi/2}+0+0\Big]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos 7\pi/2+j\sin 7\pi/2\right]+\left[\cos 7\pi+j\sin 7\pi\right)+(1+j)\left[\cos 21\pi/2+j\cdot\sin 21\pi/2\right]\right]$$

$$=\frac{1}{8}\left[5+(1-j)(0+j(-1))+(-1+j(0))+(1+j)(0+j(1))\right]$$

$$=\frac{1}{8}\left[5+(1-j)(-j)+(-1)+(1+j)(j)\right]$$

$$=\frac{1}{8}\left[5-\cancel{j}+j^2-1+\cancel{j}+j^2\right]$$

$$=\frac{1}{8}\left[4+2j^2\right]=\frac{1}{8}\left[4-2\right]=1/8\left[2\right]=1/4=0.25$$

$x(n)=\{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$

Zero Padding:-

Consider a Sequence $x(n)$ having length 'L' is given by $x(n)=\{x(0), x(1), x(2), ----x(L-1)\}$ to improve the frequency Resolution of discrete fourier transform $(2\pi/N)$. Zero padding is require that means adding no. of Zero's to the given Sequence $x(n)$.

1) find the DFT of a Sequence $x(n)=\{1$ for $0\le n\le 2$
0 otherwise
for $N=4$ & $N=8$

Sol for $N=4$

$x(n)=\{1,1,1,0\}$

$$X(k)=\sum_{n=0}^{N-1} x(n)\,e^{-j2\pi kn/4}$$

$$X(0)=\sum_{n=0}^{3} x(n)\,e^{-j2\pi(0)n/N}$$

$$X(0) = \sum_{n=0}^{3} x(n) \cdot 1$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1+1+1+0$$

$$X(0) = 3$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j2\pi(1)n/\cancel{4}2}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n/2}$$

$$= x(0)e^{-j\pi(0)/2} + x(1)e^{-j\pi/2} + x(2)e^{-j\pi(\cancel{2})/\cancel{2}} + x(3)e^{-j\pi(3)/2}$$

$$= 1 + e^{-j\pi/2} + e^{-j\pi} + 0$$

$$= 1 + [\cos\pi/2 - j\sin\pi/2] + [\cos\pi - j\sin\pi]$$

$$= 1 + [0 - j] + [-1 - j(0)]$$

$$= 1 - j - 1$$

$$= -j$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\cancel{2}\pi(2)n/\cancel{4}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= x(0)e^{-j\pi(0)} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + 1[\cos\pi - j\sin\pi] + [\cos 2\pi - j\sin 2\pi] + 0$$

$$= 1 + [-1 - j(0)] + [1 - j(0)]$$

$$= 1 - 1 + 1 = 1$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j\cancel{2}\pi(3)n/\cancel{4}2}$$

$$= \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}$$

$= x(0)e^{-j3\pi(0)/2} + x(1)e^{-j3\pi(1)/2} + x(2)e^{-j3\pi(2)/2} +$
$x(3)e^{-j3\pi(3)/2}$

$= 1 + [\cos 3\pi/2 - j\sin 3\pi/2] + [\cos 3\pi - j\sin 3\pi] + 0$

$= 1 + [0 - j(-1)] + [-1 - j(0)]$

$= \cancel{1} + j - \cancel{1}$

$= j$

$X(K) = \{3, -j, 1, j\}$

(ii) for $N=8$

$x(n) = \{1,1,1,1,0,0,0,0\}$

$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/8}$

Circular Convolution:-

The linear Convolution of two Sequences $x(n)$ of 'L' no of Samples and $h(n)$ of 'M' no of samples produce a result $y(n)$ which contains $\boxed{N=L+M-1}$ incase of circular convolution if $x(n)$ Contains 'L' no.of Samples and $h(n)$ has 'M' no of Samples then we can perform Circular Convolution b/w the two Sequence using

$$N=max(L,M)$$

To find the Circular Convolution of two Sequence their are two methods are use

1) concentric circle method
2) Matrix multiplication method

(i) concentric Circle method:-

Given two Sequences $x_1(n)$ and $x_2(n)$ the Circular convolution of these two Sequence $x_3(n)$ $x_3(n) = x_1(n) Ⓝ x_2(n)$ can be found by using the following steps.

1. Graph 'n' Samples of $x_1(n)$ as equally spaced points around an outer circle in counter clock-wise direction.

2. Start at the same point as $x_1(n)$ graph 'n' samples of $x_2(n)$ as equally Spaced points around as inner circle in clockwise direction.

3. Multiply the Corresponding Samples on the two Circles and sum the products to produce an outputs.

4. Rotate the inner circle one sample at a time in count clockwise direction and go to step3 to obtain the next value of output.

5. Repeat step4 until the inner circle first sample lines up with ~~to~~ the first sample of the exterior circle once again.

(2) Matrix Multiplication Method:-

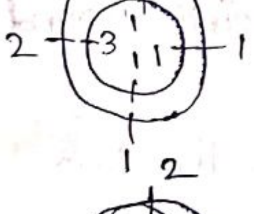
In this method, the circular convolution of two sequence $x_1(n)$ & $x_2(n)$ can be obtained by representing the sequence in matrix form that is the sequence $x_2(n)$ is repeated viva circular shift of samples and represented in NxN matrix form the sequence $x_1(n)$ is represented as column matrix. the multiplication of this two matrices gives the sequences $x_3(n)$.

1) find circular convolution of two sequence $x_1(n)=\{1,2,2,1\}$, $x_2(n)=\{1,2,3,1\}$ using concentric circle method, matrix multiplication method.

A) Given $x_1(n)=\{1,2,2,1\}$

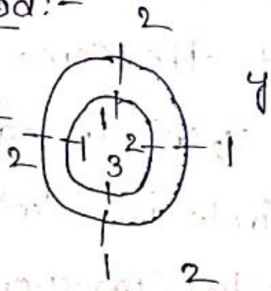
$x_2(n)=\{1,2,3,1\}$

1) Concentric circle method:-



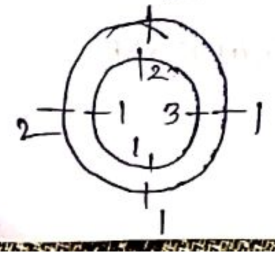
$y(0)=1+2+6+2$

$=1$



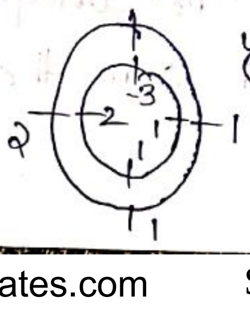
$y(1)=2+2+2+3$

$=9$



$y(2)=3+4+2+1$

$=10$



$y(3)=1+6+4+1$

$=12$

$y(n) = \{11, 9, 10, 12\}$

2) Matrix Multiplication method

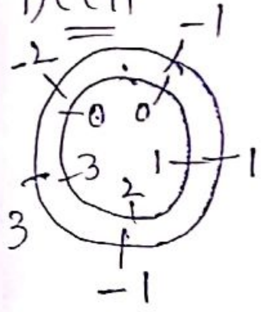
$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+6+2 \\ 2+2+3+1 \\ 2+4+3+1 \\ 1+4+6+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

2) find the Circular Convolution of two finite duration of Sequence $x_1(n) = \{1, -1, -2, 3, -1\}$ and $x_2(n) = \{1, 2, 3\}$

A) given $x_1(n) = \{1, -1, -2, 3, -1\}$

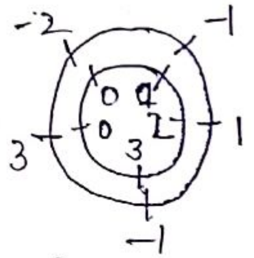
$x_2(n) = \{1, 2, 3, 0, 0\}$

1) CCM



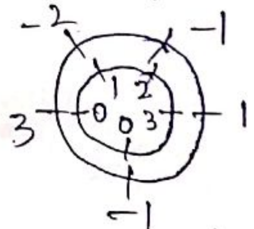
$y(0) = 1+0+9-2+0$

$= 8$



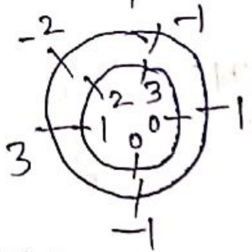
$y(1) = 2-1+0+0-3$

$= -2$

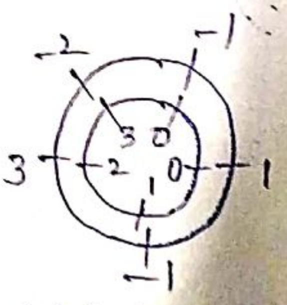


$y(2) = 3-2-2+0+0$

$= -1$



$y(3) = 0-3-4+3+0 = -4$



$y(4) = 0+0-6+6-1$

$= -1$

$y(n) = \{8, -2, -1, -4, -1\}$

2) <u>MMM</u>

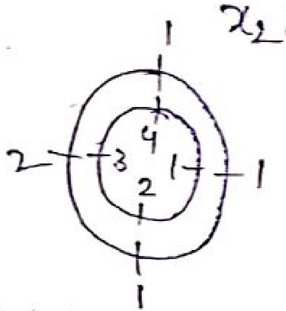
$$\begin{bmatrix} 1 & -1 & 3 & -2 & -1 \\ -1 & 1 & -1 & 3 & -2 \\ -2 & -1 & 1 & -1 & 3 \\ 3 & -2 & -1 & 1 & -1 \\ -1 & 3 & -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-2+9+0+0 \\ -1+2-3+0+0 \\ -2-2+3+0+0 \\ 3-4-3+0+0 \\ -1+6-6+0+0 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}$$

$y(n) = \{8, -2, -1, -4, -1\}$

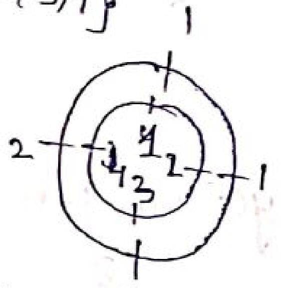
3) perform the circular convolution of the following sequence (i) $x_1(n) = \{1, 1, 2, 1\}$, $x_2(n) = \{1, 2, 3, 4\}$, (ii) $x_1(n) = \{1, 2, 3, 1\}$ and $x_2(n) = \{4, 3, 2, 2\}$

<u>Sol</u> (i) $x_1(n) = \{1, 1, 2, 1\}$

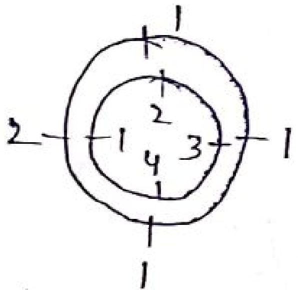
$x_2(n) = \{1, 2, 3, 4\}$



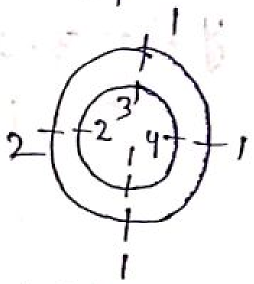
$y(0) = 1+4+6+2$
$= 13$



$y(1) = 2+1+8+3$
$= 14$



$y(2) = 3+2+2+4$
$= 11$



$y(3) = 4+3+4+1$
$= 12$

$y(n) = \{13, 14, 11, 12\}$

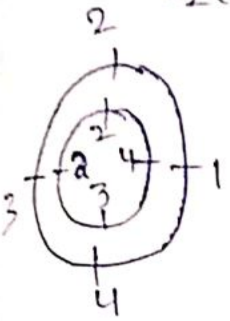
2) MMM

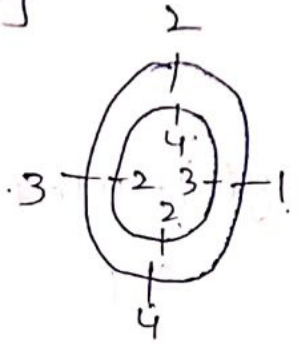
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+6+4 \\ 1+2+3+8 \\ 2+2+3+4 \\ 1+4+3+4 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

$y(n) = \{13, 14, 11, 12\}$

(ii) $x_1(n) = \{1, 2, 3, 1\}$

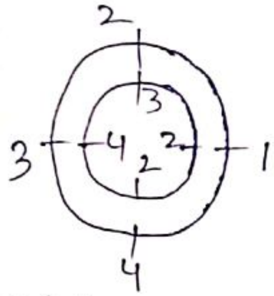
$x_2(n) = \{4, 3, 2, 2\}$

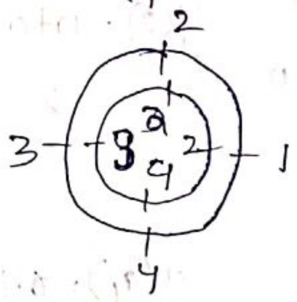




$y(0) = 4+4+6+3$
$= 17$

$y(1) = 3+8+6+8$
$= 19$





$y(2) = 2+6+12+2$
$= 22$

$y(3) = 2+4+9+4$
$= 19$

$y(n) = \{17, 19, 22, 17\}$

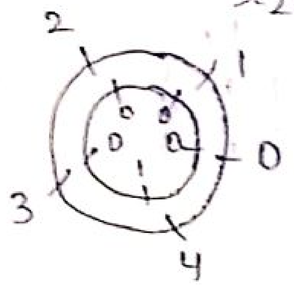
(ii)
$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+3+6+4 \\ 8+3+2+6 \\ 12+6+2+2 \\ 4+9+4+2 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

$y(n) = \{17, 19, 22, 19\}$

4) Consider the Sequence $x_1(n) = \{0, 1, 2, 3, 4\}$, $x_2(n) = \{0, 1, 0, 0, 0\}$

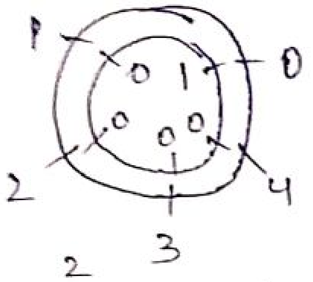
A) given $x_1(n)=\{0,1,2,3,4\}$

$x_2(n)=\{0,1,0,0,0\}$



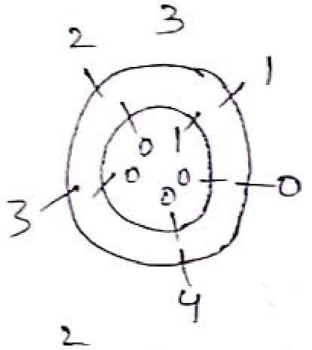
$y(0)=0+0+0+0+4$

$=4$



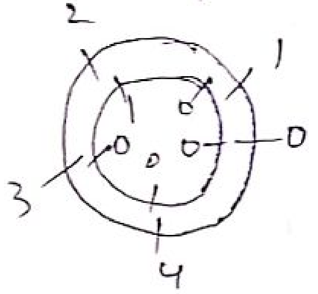
$y(1)=0+0+0+0+0$

$=0$



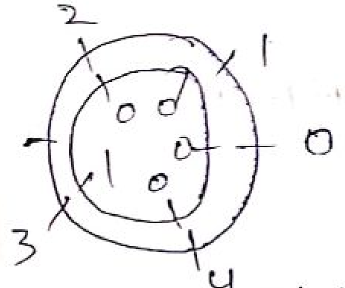
$y(2)=0+0+1+0+0$

$=1$



$y(3)=0+0+2+0+0$

$=2$



$y(4)=0+0+0+3+0$

$=3$

$y(n)=\{4,0,1,2,3\}$.

(2) MMM

$$\begin{bmatrix}0&4&3&2&1\\1&0&4&3&2\\2&1&0&4&3\\3&2&1&0&4\\4&3&2&1&0\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}0+4+0+0+0\\0+0+0+0+0\\0+1+0+0+0\\0+0+2+0+0\\0+0+0+3+0\end{bmatrix}=\begin{bmatrix}4\\0\\1\\2\\3\end{bmatrix}$$

Properties of DFT :-

1) **Linearity property :-**

If $DFT[x(n)] = X(k)$

If $DFT[x_1(n)] = X_1(k)$

$DFT[x_2(n)] = X_2(k)$ Then

$$DFT[a x_1(n) + b x_2(n)] = aX_1(k) + bX_2(k)$$

Proof:-

$$DFT[ax_1(n)+bx_2(n)] = \sum_{n=0}^{N-1}[ax_1(n)+bx_2(n)]\, e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} ax_1(n)\, e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} bx_2(n)\, e^{-j2\pi kn/N}$$

$$= a\sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N} + b\sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi kn/N}$$

$$= aX_1(k) + bX_2(k)$$

$$\boxed{\therefore\ DFT[ax_1(n)+bx_2(n)] = aX_1(k)+bx_2(k)}$$

2) **Time Shifting Property:**

If $DFT[x(n)] = X(k)$

Then $DFT[x(n-m)] = e^{-j2\pi km/N}.X(k)$

Proof:

$$DFT[x(n-m)] = \sum_{n=0}^{N-1} x(n-m)\, e^{-j2\pi kn/N}$$

let $n-m=l$

$n=l+m$

$$= \sum_{l=0}^{N-1} x(l)\, e^{-j2\pi k(l+m)/N}$$

$$= \sum_{l=0}^{N-1} x(l)\, e^{-j2\pi kl/N}.\ e^{-j2\pi km/N}$$

$$= e^{-j2\pi km/N} \sum_{l=0}^{N-1} x(l)\, e^{-j2\pi kl/N},$$

$$= e^{-j2\pi km/N}.X(k)$$

$$\boxed{\therefore\ DFT[x(n-m)] = e^{-j2\pi km/N}.X(k)}$$

rcular Convolution:-

If $DFT[x_1(n)] = X_1(K)$

$DFT[x_2(n)] = X_2(K)$

then $DFT[x_1(n) \textcircled{N} x_2(n)] = X_1(K) X_2(K)$

ultiplication of Two Sequences:

If $DFT[x_1(n)] = X_1(K)$

$DFT[x_2(n)] = X_2(K)$

Then $DFT[x_1(n) \cdot x_2(n)] = \frac{1}{N} [X_1(K) \textcircled{N} X_2(K)]$

arseval's Theorem:-

If $DFT[x(n)] = X(K)$

$DFT[y(n)] = Y(K)$

Then $\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) Y^*(K)$

Time Reversal Property:

if $DFT[x(n)] = X(K)$

Then $DFT[x(-n)] = X((-K))_N = X(N-K)$ $\qquad x((-n))_N = x(N-n)$

Proof: ~~$DFT[x(-n)] = \sum_{K=0}^{N-1} x(-n) e^{j2\pi kn/N}$~~

let $-n =$

$DFT[x((-n))_N] = DFT[x(N-n)]$

$= \sum_{n=0}^{N-1} (x(N-n) e^{j2\pi kn/N}$ $\qquad$ let $N-n=m$, $n=N-m$

$= \sum_{m=0}^{N-1} x(m) e^{j2\pi K(N-m)/N}$

$= \sum_{m=0}^{N-1} x(m) e^{j2\pi KN/N} e^{j2\pi Km/N}$

$= \sum_{m=0}^{N-1} x(m) e^{-j2\pi K} e^{j2\pi Km/N}$ $\qquad \therefore e^{j2\pi K} = 1, K=0,1,2-$

$= \sum_{m=0}^{N-1} x(m) e^{j2\pi Km/N}$

$= X((-K))_N = X(N-K)$

fast fourier Transforms:-

~~The DFT of a Sequence~~ Can

Direct evaluation of DFT:- The DFT of a sequence can be evaluated using the formula $x(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

substitution $W_N = e^{-j2\pi/N}$

$$\therefore \; X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N-1} \left[Re\, x(n) + j\, Im\, x(n)\right]\left[Re\, W_N^{kn} + j\, Im\, W_N^{kn}\right] \rightarrow ①$$

$$\Rightarrow \sum_{n=0}^{N-1} Re\, x(n)\, Re\, W_N^{kN} - \sum_{n=0}^{N-1} Im\, x(n)\, Im\, W_N^{kn} + j\left[\sum_{n=0}^{N-1} Re\, x(n)\, Im\, W_N^{kn} + \sum_{n=0}^{N-1} Re\, W_N^{k}\, Im\, x(n)\right]$$

* from eq ① we can see that to evaluate one value of x(k) the no. of complex multiplication required 'N' ∴ to evaluate all 'N' values of x(k) the no of complex multiplication is $N \times N = N^2$.

* In the same way to evaluate one value of x(k) the no of complex additions required is 'N-1' ∴ To evaluate all 'N' values of x(k) the no. of complex additions required is $N \times N-1$

* from eq ② To evaluate all values of x(k) the required no. of complex multiplications $4N^2$.

* similarly to evaluate x(k) for all values of 'k' Then required no. of complex addition required is $4N(N-1)$

* The direct evaluation of DFT is basically inefficiency because it doesn't use symmetry and periodicity properties of twiddle factor W_N.

Symmetry property → $W_N^{k+N/2} = -W_N^{k}$

Periodicity property → $W_N^{k+N} \rightarrow W_N^{k}$

Fast Fourier Transform :

The FFT algorithms Can be used the disadvantages of DFT Can be Over Come i.e

Two Basic properties of Triple factor.

Reduce the no. of complex multiplications & Addition.

FFT algorithms are based on the fundamental principle of decomposing the computation of (discrete) DFT of a sequence of a length 'N' into successively Smaller DFT

There are basically two FFT algorithms they are.

1. Decimation in Time
2. Decimation in frequency.

→ In Decimation in Time the FFT algorithm the time samples of the DFT are decomposed into smaller and ~~sm~~ small sub sequences.

→ In Decimation in frequency approach the frequency samples of the DFT are decomposed into smaller and smaller sub-sequences

1) Decimation in Time Algorithm .

This algorithm is also known as Radix -2- d which means the no of O/P points 'N' Can be expressed as a 2' i.e

let x(n) is an n point Sequence where 'n' is assumed to be a Power of '2' decimate ~~the~~ sequence into two Sequences of length N/2 where one sequence consisting of the Event Index value of x^n and the other of odd Indexed value of

The DFT of a sequence $x(n)$ is given by $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_N^{2kn} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{k(2n+1)}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{kn} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2kn} \cdot W_N^{k}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{kn} + W_N^{k} \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2kn}$$

The above equation the first sum N/2 DFT of the even indexed sequence and the second sum being the N/2. DFT of the odd indexed sequence since $X_e(k)$, and $X_o(k)$, are periodic in 'k' with period N/2.

for $k \geq N/2$ by using symmetry property

$$X(k) = X_e(k-N/2) - W_N^{k} X_o(k-N/2)$$

For $N=8$ The values of $X_e(k)$ and $X_o(k)$ are 4 point DFT's of even indexed sequence $x_e(n)$ and odd indexed sequence $x_o(n)$ respectively where

$x_e(0) = x(0)$ $\quad$ $x_o(0) = x(1)$

$x_e(1) = x(2)$ $\quad$ $x_o(1) = x(3)$

$x_e(2) = x(4)$ $\quad$ $x_o(2) = x(5)$

$x_e(3) = x(5)$ $\quad$ $x_o(3) = x(7)$

By substituting different values of 'k' i.e $k=0$ to 7

$X(0) = X_e(0) + W_8^0 X_o(0)$ $\quad$ $X(4) = X_e(0) - W_8^0 X_o(0)$

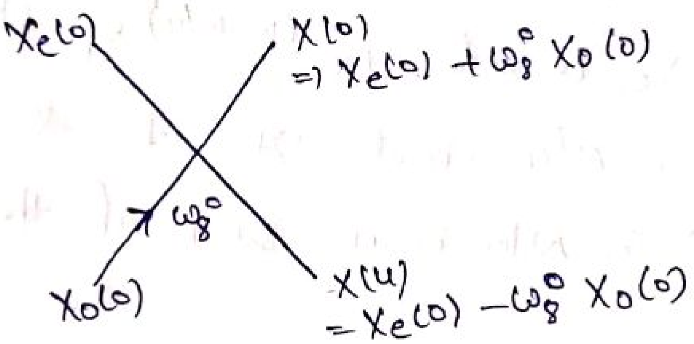
$X(1) = X_e(1) + W_8^1 X_o(1)$ $\quad$ $X(5) = X_e(1) - W_8^1 X_o(1)$

$X(2) = X_e(2) + W_8^2 X_o(2)$ $\quad$ $X(6) = X_e(2) - W_8^2 X_o(2)$

$X(3) = X_e(3) + W_8^3 X_o(3)$ $\quad$ $X(7) = X_e(3) - W_8^3 X_o(3)$

~~X(4) = Xe(4~~

from the above Set of Equations we can find that X(0) and X(4) x(1) and x(5), X(2) and X(6), X(3) and X(7) have Same inputs. X(0) is obtained by multiplying $X_o(0)$ with ω_8^0 and adding the product to $X_e(0)$. Similarly X(4) is obtained by multiplying $X_o(0)$ with ω_8^0 and subtracting the Product from $X_e(0)$. this Operation Can be represented by a "butterfly" diagram as shown in fig below.



* The 'N/2' Point DFT is Can be express as a Combination of 'N/4' Point DFT's that is $X_e(k) = X_{ee}(k) + W_N^{2k} X_{eo}(k)$

* $X_{ee}(k-N/4) - W_N^{2(k-N/4)} X_{eo}(k-N/4)$ where $X_{ee}(k)$ is 'N/4' Point DFT of the even numbers. of $x_e(n)$ and $X_{eo}(k)$ is N/4 point DFT of the odd numbers of $x_e(n)$

Similarly $X_o(k) = X_{oe}(k) + W_N^{2k} X_{oo}(k)$

$= X_{oe}(k-N/4) - W_N^{2(k-N/4)} X_{oo}(k-N/4)$

where $X_{oe}(k)$ is N/4 Point DFT of the even numbers of $x_o(n)$ and $X_{oo}(k)$ is N/4 Point DFT of the odd numbers of $x_o(n)$

* for N=8 the Sequence $x_e(n)$ Can be divided into even and odd indexed Sequences.

$x_{ee}(0) = x_e(0)$

$x_{ee}(1) = x_e(2)$

$x_{eo}(0) = x_e(1)$

$x_{eo}(1) = x_e(3)$

$X_e(0) = X_{ee}(0) + W_8^0 X_{eo}(0)$

$X_e(1) = X_{ee}(1) + W_8^2 X_{eo}(1)$

$X_e(2) = X_{ee}(0) - W_8^0 X_{eo}(0)$

$X_e(3) = X_{ee}(1) - W_8^2 X_{eo}(1)$

* Similarly the odd Sequence $X_o(n)$ is divided into even and odd indexed values.

$x_{oe}(0) = x_o(0)$

$x_{oe}(1) = x_o(2)$

$x_{oo}(0) = x_o(2)$

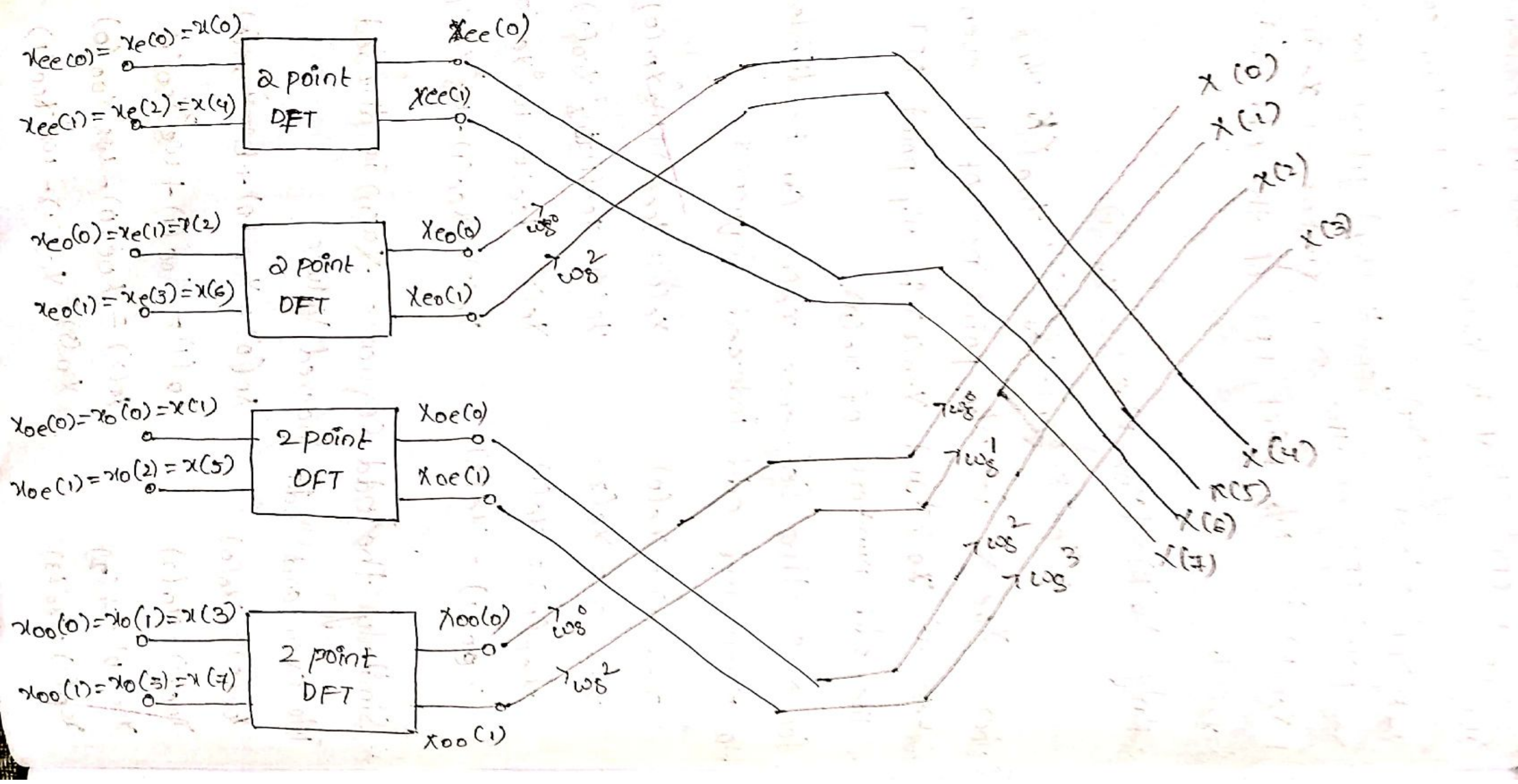
$x_{oo}(1) = x_o(3)$

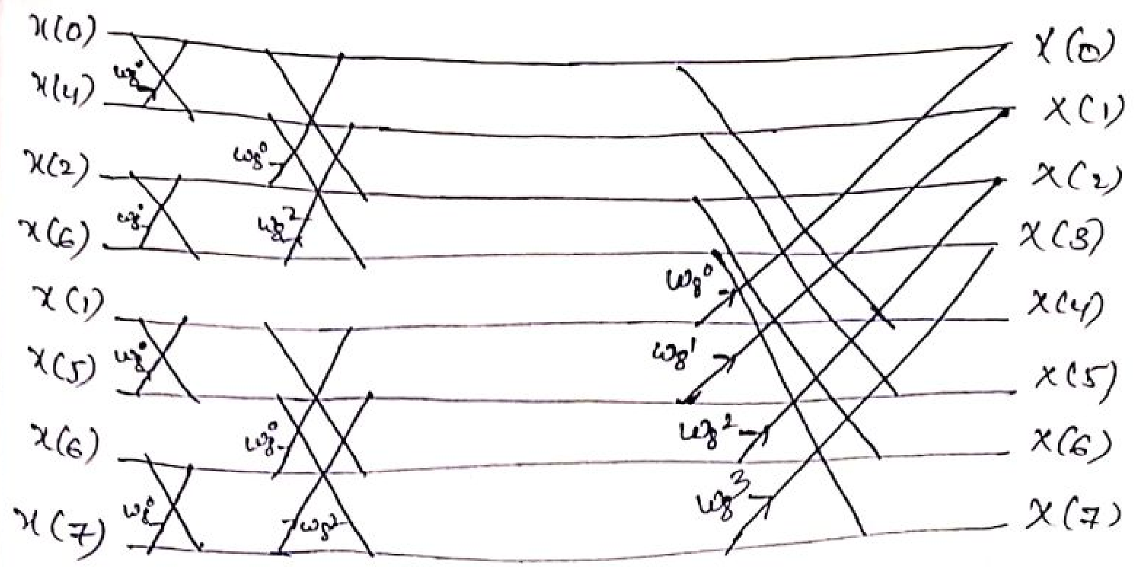
$X_o(0) = X_{oe}(0) + W_8^0 X_{oo}(0)$

$X_o(1) = X_{oe}(1) + W_8^2 X_{oo}(1)$

$X_o(2) = X_{oe}(0) - W_8^0 X_{oo}(0)$

$X_o(3) = X_{ee}(1) - W_8^2 X_{oo}(1)$


xee(0) = xe(0) = x(0)
xee(1) = xe(2) = x(4)
2 point DFT
Xee(0)
Xee(1)
xeo(0) = xe(1) = x(2)
xeo(1) = xe(3) = x(6)
2 point DFT
Xeo(0)
Xeo(1)
W8^0
W8^2
xoe(0) = xo(0) = x(1)
xoe(1) = xo(2) = x(5)
2 point DFT
Xoe(0)
Xoe(1)
xoo(0) = xo(1) = x(3)
xoo(1) = xo(3) = x(7)
2 point DFT
Xoo(0)
Xoo(1)
W8^0
W8^2
W8^0
W8^1
W8^2
W8^3
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)



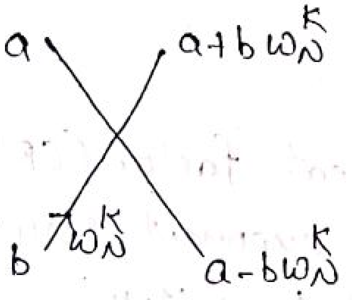
i/p Sample	Binary rep of i/p sample	Bit reversal Binary	Bit Reversal sample Index
0	0 0 0	0 0 0	0
1	0 0 1	1 0 0	4
2	0 1 0	0 1 0	2
3	0 1 1	1 1 0	6
4	1 0 0	0 0 1	8 1
5	1 0 1	1 0 1	5
6	1 1 0	0 1 1	3
7	1 1 1	1 1 1	7

In DFT Algorithm we can find that the output sequences to be in natural order that X(K) where K=0 to m-1. the input sequence has to be stored in a bit reversal order for an 8-point DFT Algorithm the input sequence is in order x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7).

for n=8 the bit Reversal process is shown in the table.

Basic operation:-

The Basic operation of DIT Algorithm in represents by using butterfly in which two inputs are combine to give two outputs at the input a and b are the given values and the output are shown in the figure.



Steps of Radix-2 Decimation in time FFT:-

1. The no of i/p samples $N=2^M$. where 'M' is an interger.

2. The i/p Sequence is shuffled through bit reversal.

3. The no of stages in the flow graph is given by $M=\log_2 N$.

4. Each stage consists of $N/2$ butterflys.

5. i/p or outputs for each butterfly are seperate by 2^{m-1}, where 'm' represents the stage index. that is for first stage $m=1$, & for second stage $m=2$, and so on.

6. The no of complex multiplications is given by $N/2 \log_2 N$.

7. The no of complex Additions is given by $N\log_2 N$.

8. The twiddle factor exponents are the function of stage 'm' index and is given by

$K = \frac{Nt}{2^m}$, $t = 0, 1, \dots 2^{m-1} - 1$

9. The no of sets of butterflyes in each stage given by 2^{M-m}

10. The exponent Repeat factor (ERF) which is the no of times the exponent sequence associated with 'm' is repeated by '2^{M-m}'

1) Find DFT of Sequence $x(n) = \{1,2,3,4,4,3,2,1\}$ using decimation in time algorithm.

Sol $x(n) = \{1,2,3,4,4,3,2,1\}$

$N = 8$

$M = \log_2 N = \log_N 2^3 = 3$

$W_8^0 = (e^{-j2\pi/N})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1$

$= e^{-j\pi/4} = \cos\pi/4 - j\sin\pi/4 = 1/\sqrt{2} - j\,1/\sqrt{2}$

$= 0.707 - 0.707j$

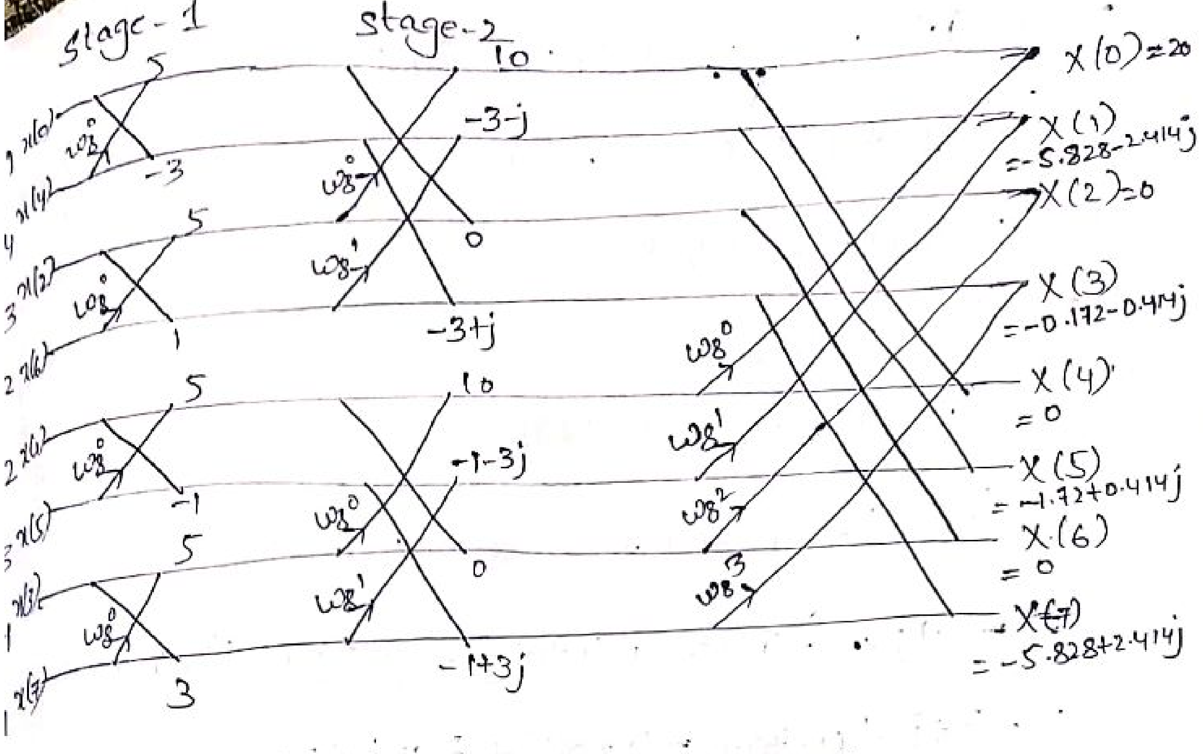
$W_8^2 = (e^{-j2\pi/8})^2$

$= e^{-j\frac{2\pi}{8_2}\cdot 2} = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$

$W_8^3 = (e^{-j2\pi/8})^3$

$= e^{-j\frac{2\pi}{8_4}\cdot 3} = e^{-j3\pi/4}$

$= \cos 3\pi/4 - j\sin 3\pi/4 = 0.707 - 0.707j$



$X(1) = (-3-j) + (0.707 - 0.707j)(-1-3j)$

$= -3-j-0.707 + j0.707 - 3\times j0.707 + 3j^2\times 0.707$

$= -3-j-0.707 + j0.707 - 2.12j - 2.12$

$= -5.83 + j(-1+0.707-2.12)$

$= -5.83 - 2.41j$

$X(5) = (-3-j) - (0.707 - j0.707)(-1-3j)$

$= -3-j-(-0.707 + j0.707 - 3\times 0.707j + 3j^2\times 0.707)$

$= -3-j+0.707 - j0.707 + 3j\times 0.707 + 3\times 0.707$

$= -3-j+0.707 - 0.707j + 2.12j + 2.12$

$= -0.17 - j(1+0.707-2.12)$

$= -0.17 + 0.41j$

$X(2) = 0 + W_8^2(0) = 0$

$X(6) = 0 - W_8^2(0) = 0$

$X(3) = (-3+j) + W_8^3(-1+3j)$

$= (-3+j) + (-0.707 + j\,0.707)(-1+3j)$

$= -3 + j + 0.707 - 0.707j - 2.12j - 2.12$

$= -4.41 + j(1 - 0.707 - 2.12)$

$= -4.41 + j(-1.83)$

$= -4.41 - 1.83j$

$X(7) = (-3+j) - W_8^3(-1+3j)$

$= (-3+j) - (-0.707 + j\,0.707)(-1+3j)$

$= -3+j - (-0.707 - 0.707j - 2.12j + 3j^2 \times 0.707)$

$= -3+j + 0.707 + 0.707j + 2.12j + 2.12$

$= -5.828 + j2.414$

$X(K) = \{20, -5.828-2.414j, 0, -0.172-j0.414, 0,$
$-1.72+j0.414, 0, -5.828+j2.414\}$

2) Compute the 8-point DFT of the sequence

$x(n) = \begin{cases} 1; & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$ using decimation in time.

Sol

$W_8^0 = (e^{-j2\pi/8})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1 = \cos \pi/4 - j\sin \pi/4 = 0.707 - j0.707$

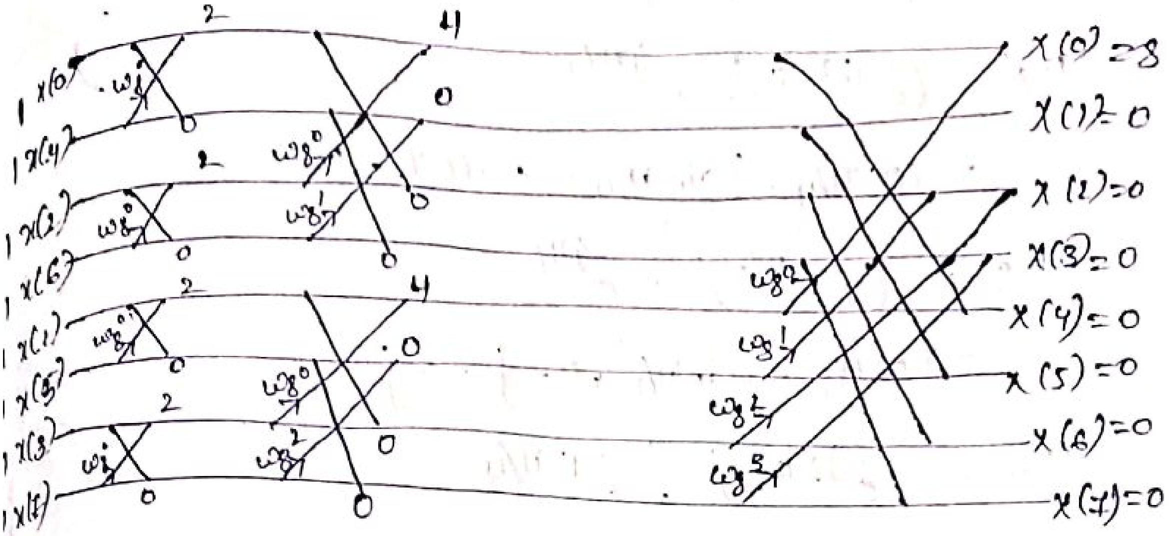
$W_8^2 = (e^{-j2\pi/8})^2 = \cos \pi/2 - j\sin \pi/2 = 0 - j = -j$

$W_8^3 = (e^{-j2\pi/8})^3 = \cos 3\pi/4 - j\sin 3\pi/4 = -0.707 - 0.707j$

$N = 8$

$M = \log_2 N = \log_2 8$

$= \log_2 2^3 = 3$

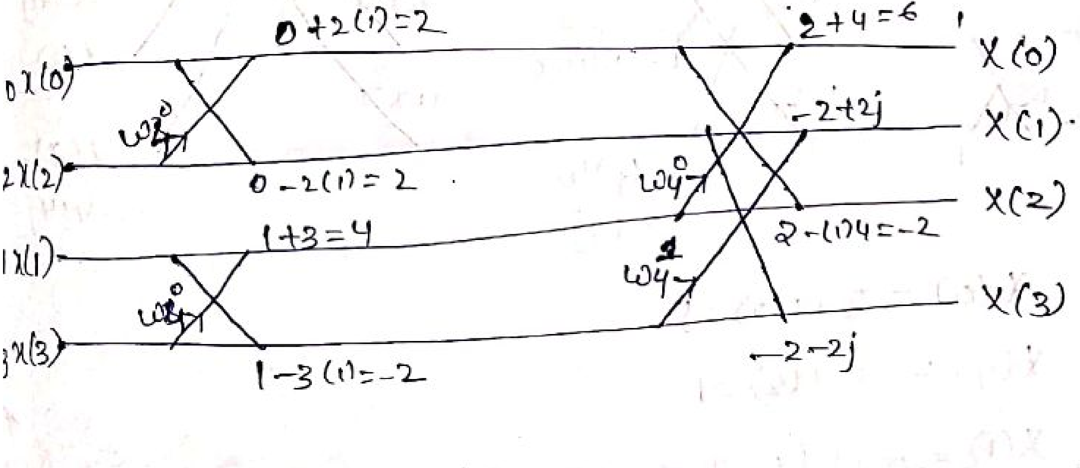


X(k) = {8, 0, 0, 0, 0, 0, 0, 0}

3) Compute 4-point DFT of the sequence $x(n) = \{0, 1, 2, 3\}$ using decimation in time Algorithm.

$W_4^0 = (e^{-j2\pi/4})^0 = 1$

$W_4^1 = (e^{-j2\pi/4})^1 = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$



X(k) = {6, -2+2j, -2, -2-2j}

4) find 8-point DFT of Sequence $x(n) = \{1, 1, 1, 1, 1, 0, 0, 0\}$ using Decimation in time algorithm.

Sol $w_8^0 = (e^{-j2\pi/N})^0 = 1$

$w_8^1 = (e^{-j2\pi/8})^1 = e^{-j\pi/4}$

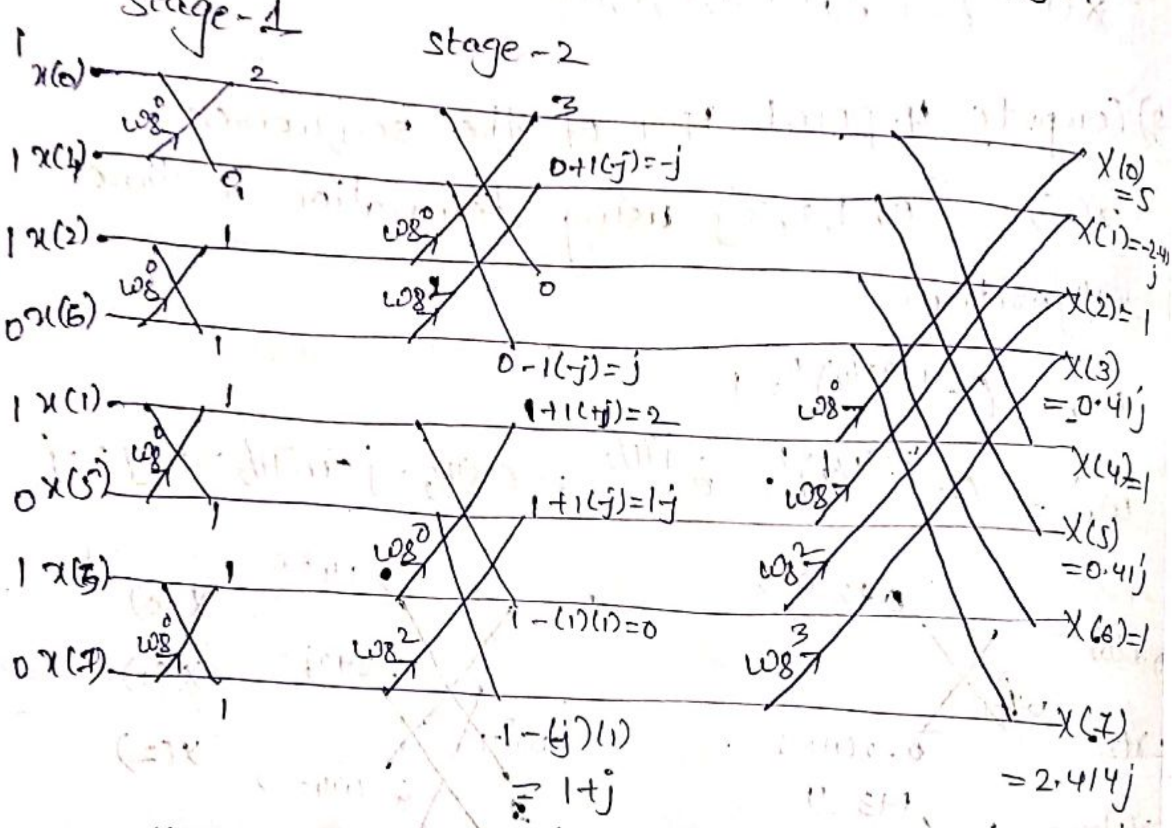
$= \cos\pi/4 - j\sin\pi/4 = 0.707 - 0.707j$

$w_8^2 = (e^{-j2\pi/8})^2 = e^{-j\pi/2}$

$= \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$

$w_8^3 = (e^{-j2\pi/8})^3 = e^{-j3\pi/4}$

$= \cos3\pi/4 - j\sin3\pi/4 = -0.707 - 0.707j$



$X(0) = 3+1(2) = 5$

$X(4) = 3-1(2) = 1$

$X(1) = -j + (0.707 - j0.707)(1-j)$

$= -j + (0.707 - j0.707 - 0.707j + j^2 0.707)$

$= -j + \cancel{0.707} - 1.41j - \cancel{0.707}$

$= -2.41j$

$$X(5) = -j - (0.707 - 0.707j)(1j)$$

$$= -j - (0.707 - 0.707j - 0.707j + 0.707j^2)$$

$$= -j - (\cancel{0.707} - 1.41j - \cancel{0.707})$$

$$= -j + 1.41j \quad = 0.41j$$

$$X(2) = 1 + (j)(0) = 1$$

$$X(6) = 1 - (j)(0) = 1$$

$$X(3) = j + (-0.707 - 0.707j)(1+j)$$

$$= j + (-0.707 - 0.707j - 0.707j - j^2 0.707)$$

$$= j + (\cancel{-0.707} - 1.41j + \cancel{0.707j})$$

$$= j - 1.41j \quad = -0.41j$$

$$X(7) = j - (-0.707 - j0.707)(1+j)$$

$$= j - (-0.707 - 0.707j - 0.707j - j^2 0.707)$$

$$= j - (\cancel{-0.707} - 1.41j + \cancel{0.707})$$

$$= j + 1.41j$$

$$= 2.414j$$

$$X(k) = \{5, -2.41j, 1, -0.4j, 1, 0.414j, 1, 2.414j\}$$

⑤ Draw the gt flow graph & the 16 point DFT and FFT

No. of Input Samples is 16

$N = 16 = 0^4$

→ The i/p Sequence bit Reversal

$N = 16 = 2^4$

0 - 0000 → 0000 - 0

1 - 0001 → 1000 - 8

2 - 0010 → 0100 - 4

3 - 0011 → 1100 - 12

4 - 0100 → 0010 - 2

5 - 0101 → 1010 - 10

6 - 0110 → 0110 - 6

7 - 0111 → 1110 - 14

8 - 1000 → 0001 - 1

9 - 1001 → 1001 - 9

10 - 1010 → 0101 - 5

11 - 1011 → 1101 - 13

12 - 1100 → 0011 - 3

13 - 1101 → 1011 - 11

14 - 1110 → 0111 → 7

15 - 1111 → 1111 → 15

Steps

No of Stages $M = \log_2^N$

$= \log_2^{16} = 4.$

Each Stage have $\frac{N}{2}$ butterflies $\Rightarrow \frac{16}{2} = 8$

Spacing b/w butterflies = $2^{m-1} = 2^0 = 1$

→ No of complex multiplications - $N/2 \log_2^N$

$= \frac{16}{2} \log_2^{16} = 8 \log_2^{16}$

$= 8 \log_2^{2^4}$

$= 8 \times 4 \log_2^2 = 32.$

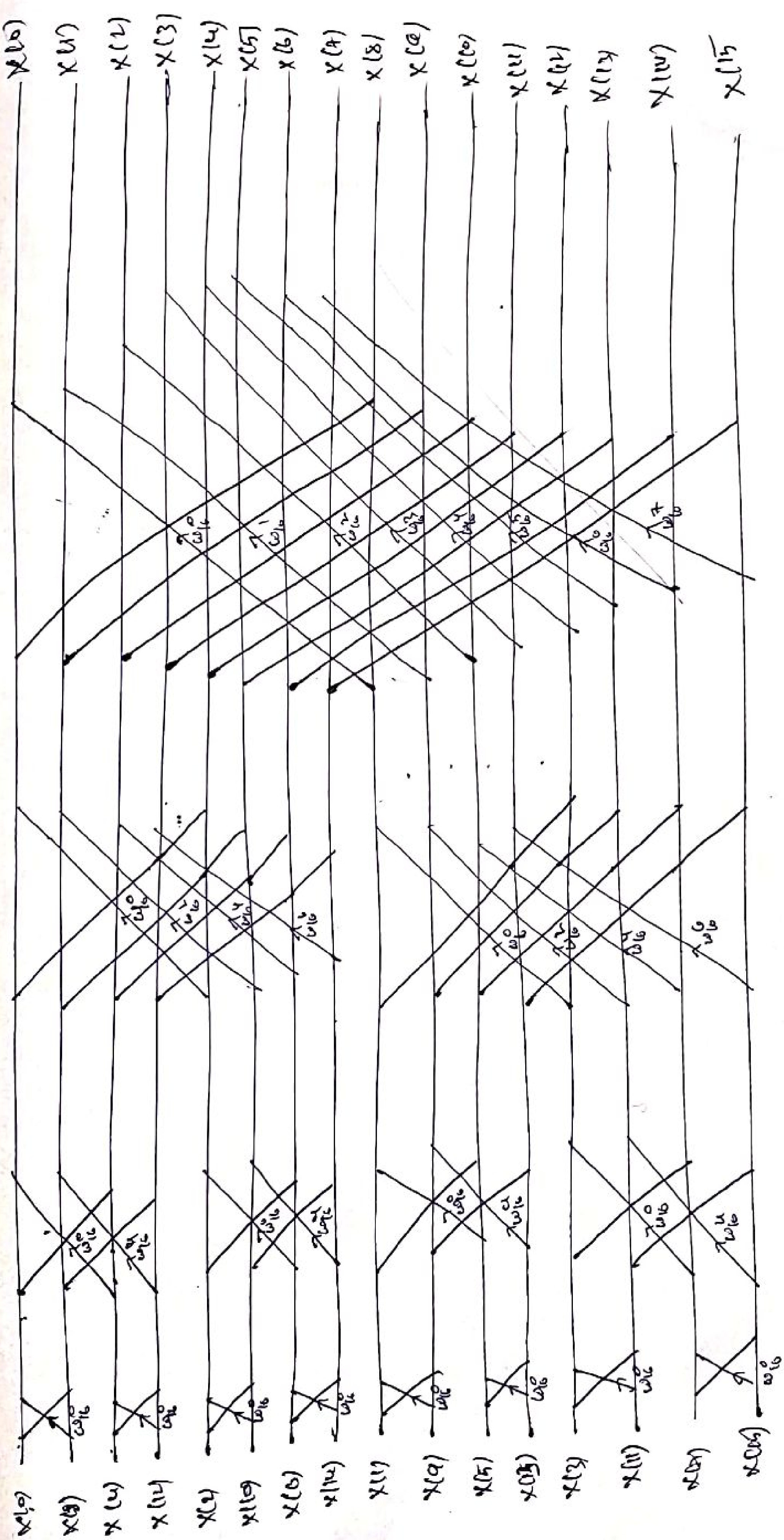
→ Twiddle factor

$K = \frac{Nt}{2m}$ $t \to 0 \text{ to } 2^{M-1} - 1$ $m = 1$

$0 \text{ to } 2^{0-1} - 1$

$= \frac{16(0)}{2}$ $= 0 \text{ to } 0$

$K = 0$


x(0)
x(8)
x(4)
x(12)
x(2)
x(10)
x(6)
x(14)
x(1)
x(9)
x(5)
x(13)
x(3)
x(11)
x(7)
x(15)
W_{16}^0
W_{16}^0
W_{16}^0
W_{16}^0
W_{16}^0
W_{16}^0
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W_{16}^0
W_{16}^0
W_{16}^4
W_{16}^0
W_{16}^4
W_{16}^0
W_{16}^4
W_{16}^0
W_{16}^4
W_{16}^0
W_{16}^2
W_{16}^4
W_{16}^6
W_{16}^0
W_{16}^2
W_{16}^4
W_{16}^6
W_{16}^0
W_{16}^1
W_{16}^2
W_{16}^3
W_{16}^4
W_{16}^5
W_{16}^6
W_{16}^7
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
X(8)
X(9)
X(10)
X(11)
X(12)
X(13)
X(14)
X(15)

~~Decimation in~~ Frequency Fast Fourier Transform Algorithm:-

In Decimation in frequency Algorithm the output Sequence X(K) is divided into th smaller and Smaller Sub Sequence.

In this Algorithm the input sequence x(n) is partion into two Sequence each of length N/2 Samples. the first Sequence $x_1(n)$ consists of first N/2 Samples of x(n). and Second Sequence $x_2(n)$ Consists of the last N/2 Samples of x(n).

i.e., $x_1(n) = x(n)$ for $n = 0, 1, \ldots N/2 - 1$

$x_2(n) = x(n + N/2)$ for $n = 0, 1, \ldots N/2 - 1$

The N-point DFT of x(n) Can be written as

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(n)\, W_N^{kn} + \sum_{n=N/2}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{K(n+N/2)}$$

$$= \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + W_N^{KN/2} \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{kn}$$

$$W_N^{KN/2} = e^{-j\frac{2\pi}{\cancel{N}} K \frac{\cancel{N}}{\cancel{2}}}$$

$$= e^{-j\pi K}$$

$$X(K) = \sum_{n=0}^{N/2-1} x_1(n) W_N^{kn} + e^{-j\pi k} \sum_{n=0}^{N/2-1} x_2(n) W_N^{kn}$$

for Even Values of k $e^{-j\pi k} = 1$

$$\therefore X(2K) = \sum_{n=0}^{N/2-1} x_1(n) W_N^{kn} + \sum_{n=0}^{N/2-1} x_2(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} [x_1(n) + x_2(n)] W_N^{2kn}$$

$$= \sum_{n=0}^{N/2-1} f(n) W_N^{2kn}$$

where $f(n) = x_1(n) + x_2(n)$

* f(n) is obtained by adding the first half and last half of the input Sequence x(n).

when k is odd $e^{-j\pi k} = -1$

$$\therefore X(2K+1) = \sum_{n=0}^{N/2-1} x_1(n) W_N^{kn} - \sum_{n=0}^{N/2-1} x_2(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} [x_1(n) - x_2(n)] W_N^{(2K+1)n}$$

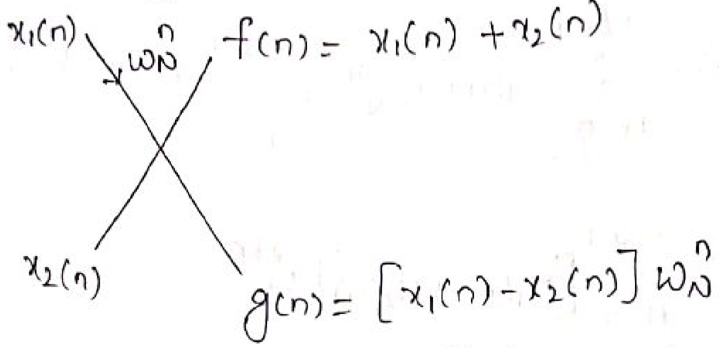
$$= \sum_{n=0}^{N/2-1} [x_1(n) - x_2(n)] W_N^{n} \, W_N^{2kn}$$

$$= \sum_{n=0}^{N/2-1} g(n) W_N^{2kn}$$

where $g(n) = [x_1(n) - x_2(n)] W_N^n$

*g(n) is obtained the Subtracting the Second half of Sequence from first half of the input Sequence and that multiplying (results with) relevant Sequence W_N^{n}.

from the above two sequences of $f(n)$ and $g(n)$ the basic operation can be represented by butterfly as shown in figure.



$f(n) = x_1(n) + x_2(n)$

$g(n) = [x_1(n) - x_2(n)] W_N^n$

For $n=8$ we have $x(0) = \sum_{n=0}^{N/2-1} f(n) W_N^{2kn}$

$$X(0) = \sum_{n=0}^{3} f(n) W_8^{2(0)n}$$

$$= \sum_{n=0}^{3} f(n) \cdot 1$$

$$= f(0) + f(1) + f(2) + f(3)$$

$$X(2) = \sum_{n=0}^{3} f(n) \cdot W_8^{2(1)n}$$

$$= \sum_{n=0}^{3} f(n) W_8^{2n}$$

$$= f(0) W_8^{2(0)} + f(1) W_8^{2(1)} + f(2) W_8^{2(2)} + f(3) W_8^{2(3)}$$

$$= f(0) + f(1) W_8^2 + f(2) W_8^4 + f(3) W_8^6$$

$W_8^4 = -1$

$W_8^8 = 1$

$W_8^6 = -W_8^2$

$$X(2) = f(0) + f(1) W_8^2 - \cancel{W_8} f(2) - f(3) W_8^2$$

$$X(4) = \sum_{n=0}^{3} f(n) W_8^{2(2)n}$$

$$X(4) = \sum_{n=0}^{3} f(n)\, w_8^{4n}$$

$$= \sum_{n=0}^{3} f(n)\, (-1)^n$$

$$= f(0)(-1)^0 + f(1)(-1)^1 + f(2)(-1)^2 + f(3)(-1)^3$$

$$X(4) = f(0) - f(1) + f(2) - f(3)$$

$$X(6) = \sum_{n=0}^{3} f(n)\, w_8^{2(3)n}$$

$$= \sum_{n=0}^{3} f(n)\, w_8^{6n}$$

$$= \sum_{n=0}^{3} f(n)\, (-w_8^2)^n$$

$$= f(0)(-w_8^2)^0 + f(1)(-w_8^2)^1 + f(2)(-w_8^2)^2 + f(3)(-w_8^2)^3$$

$$= f(0) - f(1)w_8^2 - f(2) + f(3)\, w_8^2$$

$$X(1) = \sum_{n=0}^{N/2-1} g(n)\, w_N^{2kn}$$

$$= \sum_{n=0}^{3} g(n)\, w_8^{2(0)n}$$

$$= \sum_{n=0}^{3} g(n) \cdot 1$$

$$= g(0) + g(1) + g(2) + g(3)$$

$$X(3) = \sum_{n=0}^{3} g(n)\, w_8^{2(1)n}$$

$$= \sum_{n=0}^{3} g(n)\, w_8^{2n}$$

$$= \sum_{n=0} g(0)\, w_8^{2(0)} + g(1)\, w_8^{2(1)} + g(2)\, w_8^{2(2)} + g(3)\, w_8^{2(3)}$$

$$X(3) = g(0) + g(1)\, W_8^2 + g(2)\, W_8^4 + g(3)\, W_8^6$$

$$X(3) = g(0) + g(1)\, W_8^2 - g(2) - g(3)\, W_8^2$$

$$X(5) = \sum_{n=0}^{3} g(n)\, W_N^{2(2)n}$$

$$= \sum_{n=0}^{3} g(n)\, W_8^{4n}$$

$$= \sum_{n=0}^{3} g(n)\, (-1)^n$$

$$= g(0)(-1)^0 + g(1)(-1)^1 + g(2)(-1)^2 + g(3)(-1)^3$$

$$= g(0) - g(1) + g(2) - g(3)$$

$$X(7) = \sum_{n=0}^{3} g(n)\, W_N^{2(3)n}$$

$$= \sum_{n=0}^{3} g(n)\, W_8^{6n}$$

$$= \sum_{n=0}^{3} g(n)(-W_8^2)^n$$

$$= g(0)(-W_8^2)^0 + g(1)(-W_8^2)^1 + g(2)(-W_8^2)^2 +$$

$$g(3)(-W_8^2)^3$$

$$= g(0) - g(1)\, W_8^2 + g(2)(-1) - g(3)(W_8^6)$$

$$= g(0) - g(1)\, W_8^2 - g(2) + g(3)\, W_8^2$$

We have Seen that the even indexed Samples of $X(k)$ Can be obtained from the four point DFT of the Sequences $f(n)$. where $f(n) = x_1(n) + x_2(n)$.

$f(0) = x_1(0) + x_2(0)$

$f(1) = x_1(1) + x_2(1)$

$f(2) = x_1(2) + x_2(2)$

$f(3) = x_1(3) + x_2(3)$

The odd indexed values of X(K) can be obtain from the 4-point DFT of sequence g(n).

$g(n) = [x_1(n) - x_2(n)] W_N^n$

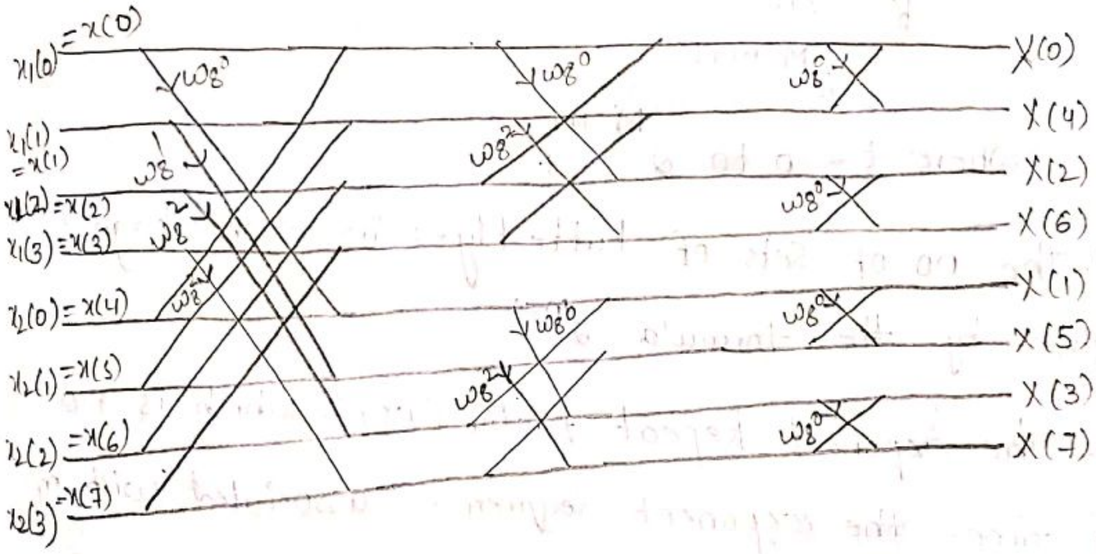
$g(n) = [x_1(n) - x_2(n)] W_8^n$

$g(0) = [x_1(0) - x_2(0)] W_8^0$

$g(1) = [x_1(1) - x_2(1)] W_8^1$

$g(2) = [x_1(2) - x_2(2)] W_8^2$

$g(3) = [x_1(3) - x_2(3)] W_8^3$



Steps for Radix-2 Decimation in Frequency FFT Algorithm:-

1. The no of input samples $N = 2^M$, where M is the no of stages.
2. The input sequence is in natural order.
3. The no of stages in the flow graph is given by $M = \log_2 N$

4. Each stage consits of $N/2$ butterflyes.

5. inputs (or) output for each butterfly are seperated by 2^{M-m} Samples.

where 'm' represents the stage index that is for first stage $M=1$ and Second

6. The no of complex multiplications is given by $N/2 \log_2 N$.

7. The no of Complex Additions is given by $N\log_2 N$

8. The twiddle factor Components (or) a function of Stage index and is given by

$$K = \frac{Nt}{2^{M-m+1}}$$

Where $t = 0$ to $2^{M-m}-1$

9. The no of sets of butterflyes in each stage is given by the formula 2^{M-1}

10. The exponent Repeat factor (ERF) which is no of times the exponent Sequence associated with 'm' repeated is given by 2^{m-1}

Difference And Similarities Between DIT and DIF

Difference

1. For Decimation in time the input is bit reversal while the output is natural order. where as for Decimation in frequency the i/p is in natural order while the output is bit Reversal order.

2. The DIF butterfly is Silightly different from DIT where in DIF the complex multiplication takes place after the add Subtract operation.

Similarities:-

1. Both Algorithms require $N \log_2 N$ operations to calculate the DFT.

2. Both Algorithms can be done in place and both need to perform bit Reversal at some place during the calculation

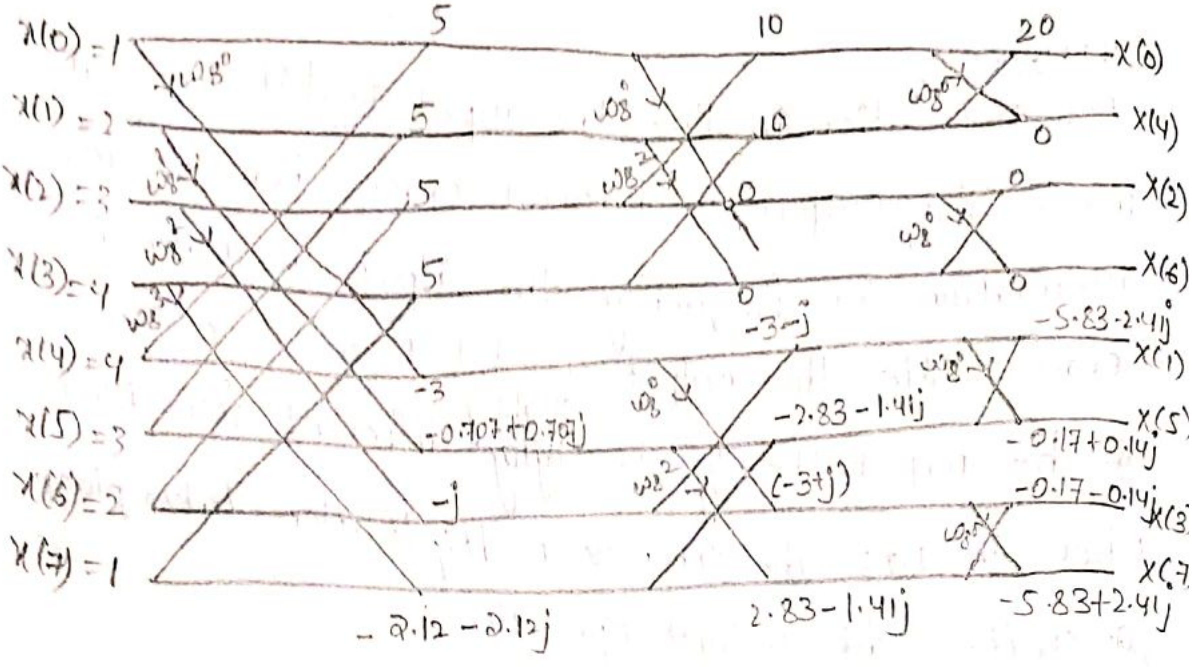
1) Find the DFT of the Sequence $x(n)=\{1,2,3,4,4,3,2,1\}$ using Decimation in frequency Algorithm.

Sol given $W_8^0 = (e^{-j2\pi/8})^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



$(-3+j)(1)$

$-0.707+0.707j - 2.12 - 2.12j$

$-0.707+0.707j + 2.12 + 2.12j$

$(1.41 + 2.83j)(-j)$

$-1.41j + 2.83$

$-3-j - 2.83 - 1.41j$

$-5.83 - 2.41j$

$-3-j + 2.83 + 1.41j$

$(0.17 + 0.41j)$

$-3+j + 2.83 - 1.41j$

$-0.17 - 0.41j$

$(-3+j - 2.83 + 1.41j)$

$X(K) = \{20, -5.83 - 2.41j, 0, -0.17-0.14j, 0, -0.17+0.14j,$
$0, -5.83+2.41j\}$

2) Find the Calculate the 8-point DFT of the Sequence $x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$ by using Decimation in time & Decimation in frequency Algorithms.

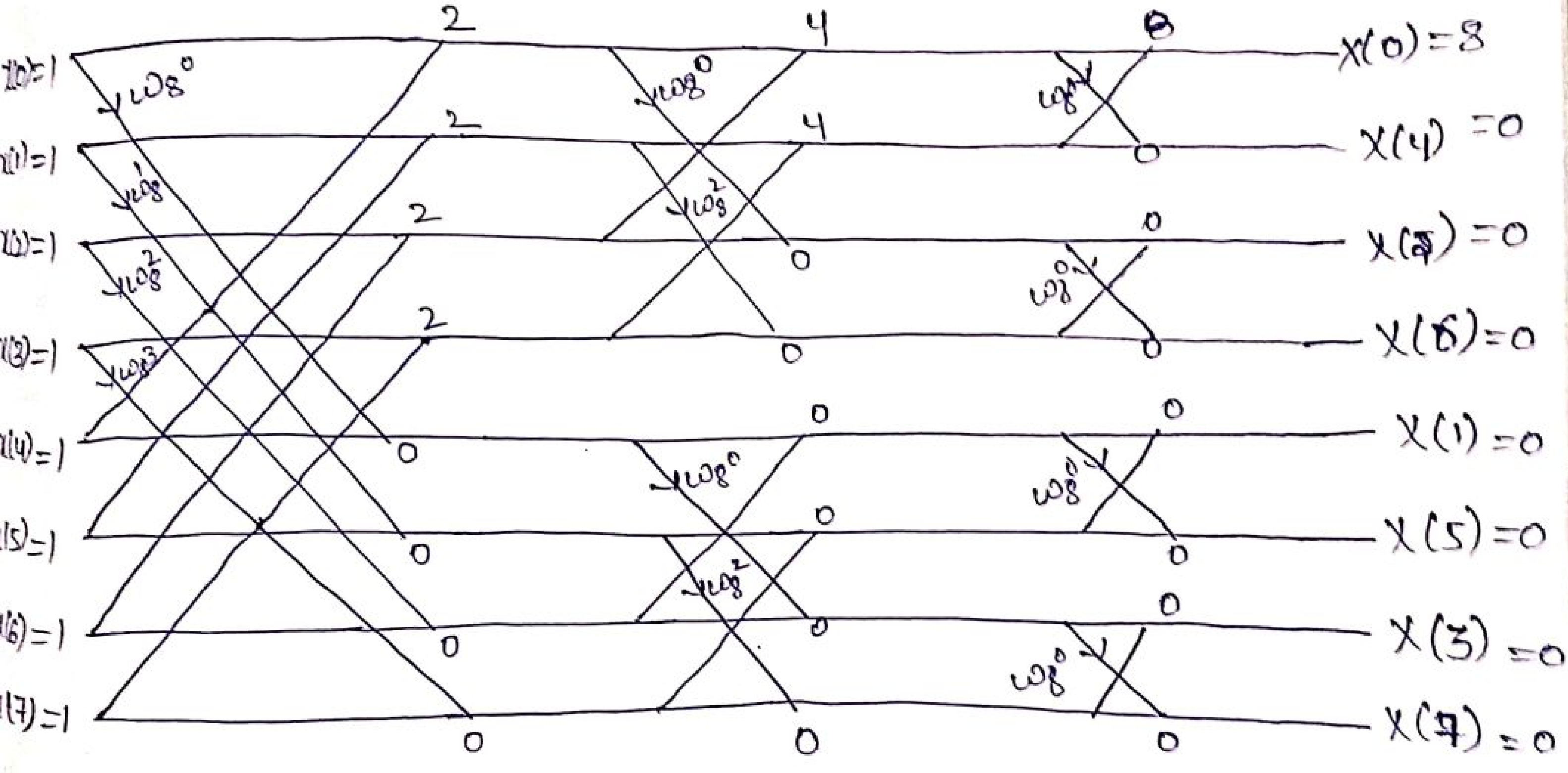
Sol) Decimation in Frequency

$w_8^0 = 1$

$w_8^1 = 0.707 - j0.707$

$w_8^2 = -j$

$w_8^3 = -0.707 - j0.707$



X(K) = {8, 0, 0, 0, 0, 0, 0}

21/12/17

Unit : 3

i) Review of Z - Transforms

2. Realization of Digital filter structures:

Digital filters can be realized by the following two realization techniques:

1. Recursive Realization. (or) IIR Digital filter structures.
2. Non - Recursive Realization (or) FIR Digital filter structures.

1. Recursive Realization:

for recursive realization the current o/p $y(n)$ is a function of present i/p, past i/p and past o/p samples.

$$y(n) = \{x(n), x(n-1), x(n-2), --- \& y(n-1), y(n-2), -- ---\}$$

This realization corresponds to IIR Digital filter structures.

2. Non Recursive Realization:

for Non Recursive realization, the current o/p $y(n)$ is a function present & past i/p samples only. This realization corresponds to FIR Digital filter structures.

$$y(n) = \{x(n), x(n-1), x(n-2), x(n-3) ---\}$$

IIR Digital filter structures (or) Recursive Realization techniques:

There are seven well known to realize IIR Digital filter.

i) Direct form - I
ii) Direct form - II & canonic (or) min. no. of memory ~~iii~~ locations required realization technique.
iii) Transposed Realization.
iv) ~~parallel~~ cascade Realization.
v) parallel Realization.
vi) Lattice Realization
vii) ladder Realization.

i) Direct form-I Realization technique;

Let us consider a system (IIR Digital filter) described by a N^{th} order LCCDE is given by (linear constant coefficient Difference eqn)

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) \quad — ①$$

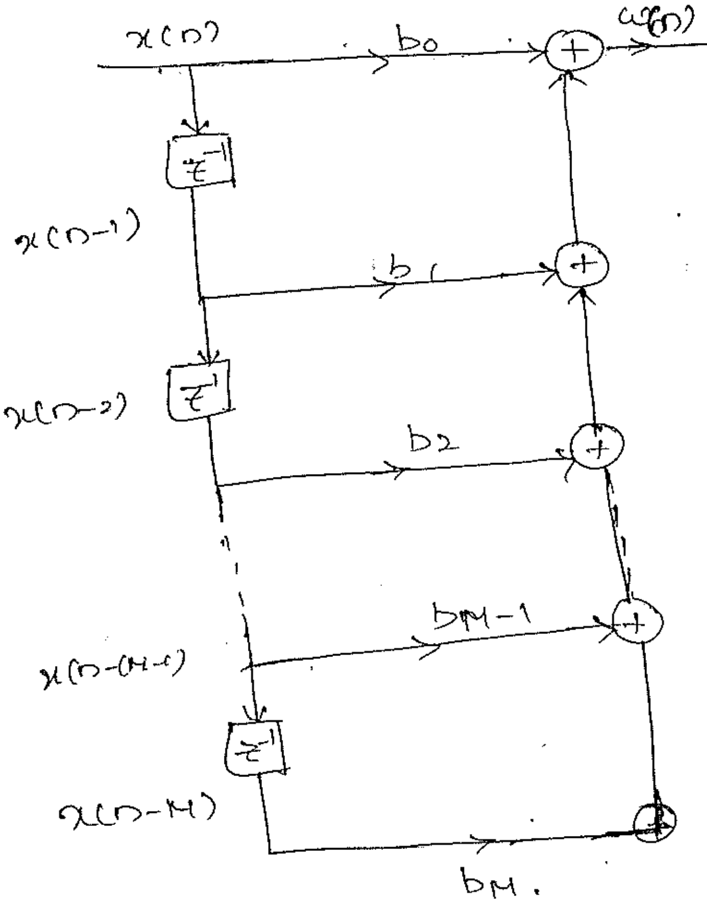
$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \cdots + a_{N-1} y(n-(N-1)) - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_{M-1} x(n-(M-1)) + b_M x(n-M) \quad — ②$$

consider
$$w(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_{M-1} x(n-(M-1)) + b_M x(n-M) \quad — ③$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \dots - a_{N-1} y(n-(N-1))$$

$$-a_N y(n-N) + w(n) \text{ ——— } ④ .$$

Eqn ③ can be realized by using direct form I realization as shown in fig(a).



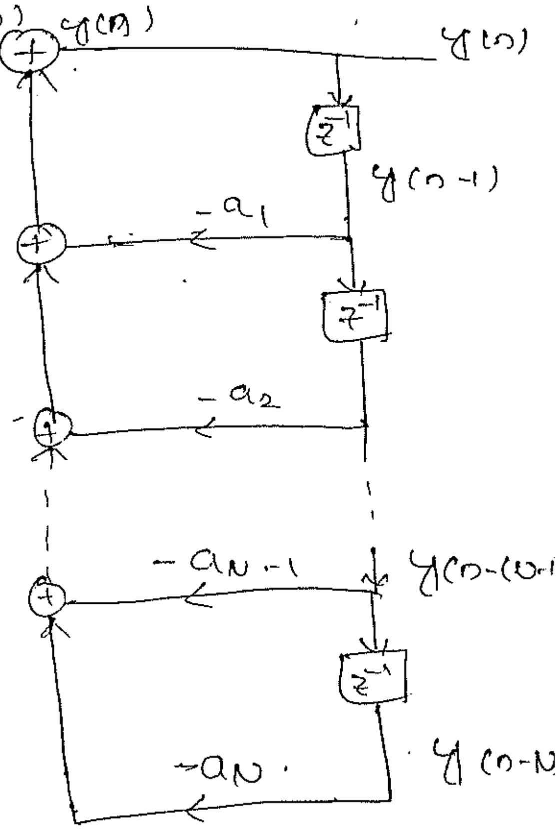


fig b.

fig(a).

fig c : Direct Form-I structure

Eqn ④ can be realized as shown in fig(b).

The above structure is called Direct form - I IIR Digital filter structure, which used separate delays for both input and output as shown in above fig c.

This realization requires total no. of multiplications = M+N+1.

The total no.of addition required = M+N.

The total no.of memory locations required = M+N.

Direct Form II (or) canonic (or) Minimum no.of memory locations realization:

let us consider a system (IIR Digital filter) described by N^{th} order LCCDE

$$y(n) = -\sum_{K=1}^{N} a_K y(n-K) + \sum_{K=0}^{M} b_K x(n-K) \quad —①$$

Apply z - Transforms.

$$Y(z) = -\sum_{K=1}^{N} a_K Y(z) z^{-K} + \sum_{K=0}^{M} b_K X(z) z^{-K}.$$

$$Y(z)\left[1+\sum_{K=1}^{N} a_K z^{-K}\right] = \sum_{K=0}^{M} b_K X(z) z^{-k}.$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{K=0}^{M} b_K z^{-K}}{1+\sum_{K=1}^{N} a_K z^{-K}}.$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = \sum_{K=0}^{N} b_K z^{-K}$$

$$Y(z) = W(z)\left[b_0 z^{0} + b_1 z^{-1} + b_2 z^{-2} + - - -\right]$$

$$y(n) = w(n)b_0 + w(n-1)b_1 + - - - \quad ②$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\sum_{K=1}^{N} a_K z^{-K}}.$$

1+N.

+N.

$$W(z)\left[1+\sum_{k=1}^{N}a_k z^{-k}\right]=X(z).$$

$\omega(z)$ ~~$+x(z)$~~

$$\omega(z)+\sum_{k=1}^{N}a_k z^{-k}\omega(z)=X(z).$$

org.

Apply I.Z.T, we get

$$I.Z.T\left[\omega(z)+\sum_{k=1}^{N}a_k z^{-k}\omega(z)\right]=IZT\left[X(z)\right]$$

By linearity property of Z.T, we get

$$IZT\left[\omega(z)\right]+\sum_{k=1}^{N}a_k\, IZT\left[z^{-k}\omega(z)\right]=IZT\left[X(z)\right]$$

we know:

$$x(n)\xleftrightarrow{Z.T}X(z)$$

$$\omega(n)\longleftrightarrow W(z)$$

$$\omega(n-k)\longleftrightarrow z^{-k}\omega(z)$$

$$y(n)\longleftrightarrow Y(z).$$

$$\omega(n)+\sum_{k=1}^{N}a_k\,\omega(n-k)=x(n).$$

$$\omega(n)=x(n)-\sum_{k=1}^{N}a_k\,\omega(n-k).$$

$$\omega(n)=x(n)-a_1\omega(n-1)-a_2\omega(n-1)------a_{N-1}$$

$$\omega(n-(N-1))-a_N\,\omega(n-N)\text{ ——— ①}.$$

$$\frac{Y(z)}{\omega(z)}=\sum_{k=0}^{M}b_k z^{-k}.$$

$$Y(z)=\sum_{k=0}^{M}b_k z^{-k}\omega(z).$$

Apply I.Z.T & linearity property.

$$I.Z.T\left[Y(z)\right] = \sum_{k=0}^{M} b_k \, I.Z.T\left[z^{-k} W(z)\right]$$

$$y(n) = \sum_{k=0}^{M} b_k \, w(n-k).$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \cdots + b_{m-1} w(n-(m-1)) + b_m w(n-m). \quad \text{— (ii)}$$

eqn ① can be realized using Direct form-II structure as shown in below fig a

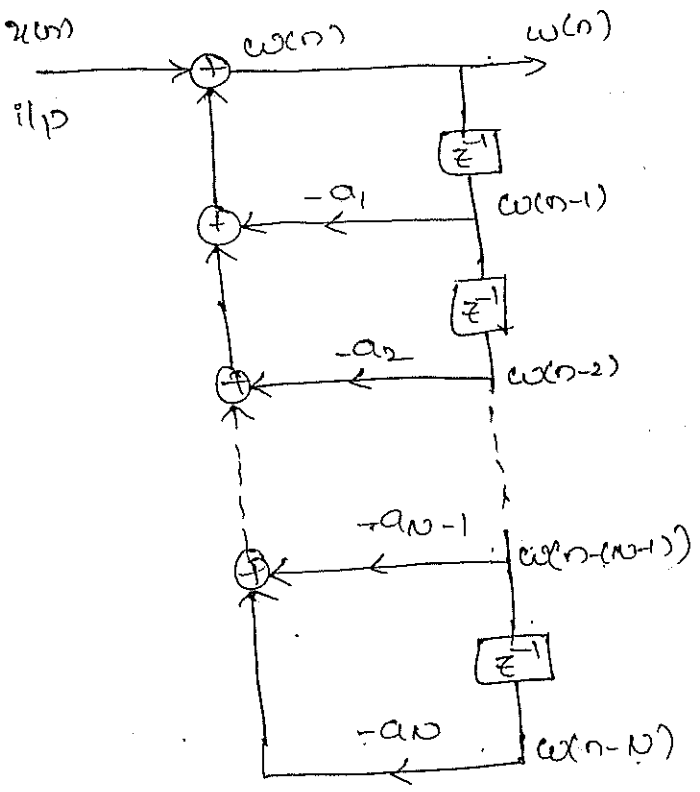
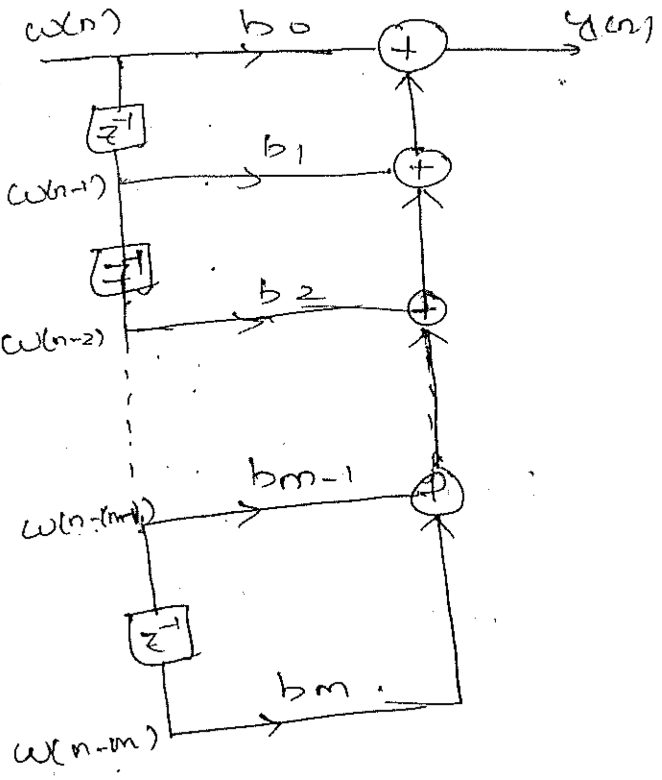
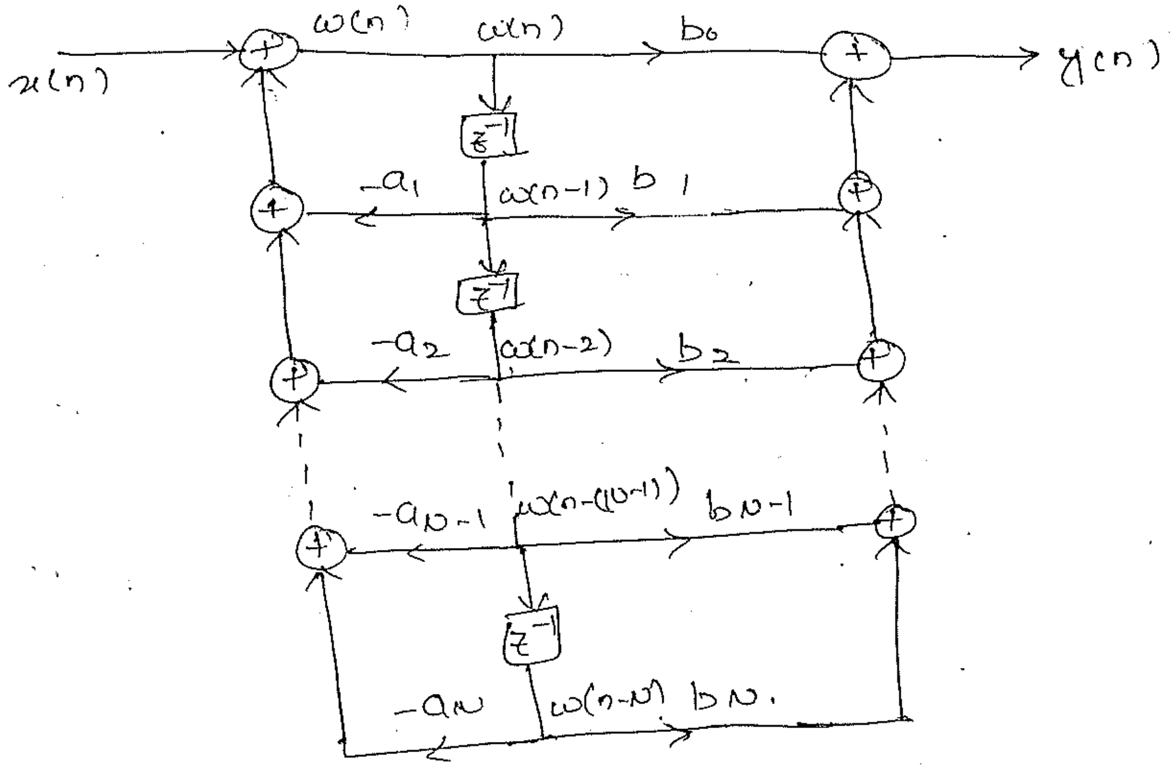


fig a



Eqn ② can be realized as show in fig b.

for N=M



~~N~~th order ~~LCCDE~~

fig (a): Direct form-II structure for Nth order LCCDE.

The above structure is called Direct form-II structure, which used common delay factors both i/p & o/p side. By using this realization, we can reduce half of delay terms (or) memory locations compare with direct form-I structure. Therefore this realization is also known as canonic (or) min. no. of memory locations required realization. For this realization, the total no. of multiplier required = M+N.

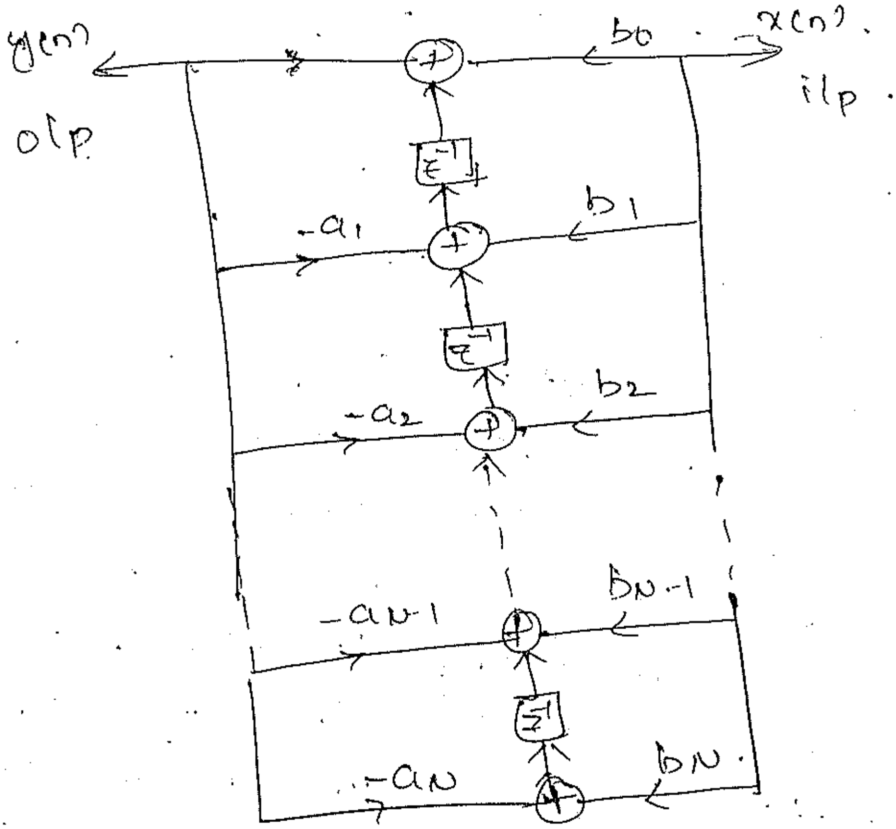
total no. of adders = M+N.

total no. of memory locations = max(M, N).

(iii) Transposed structure:

Procedure steps:

1. from the given difference eqn, we have to implement Direct form - II structure.
2. Interchange i/ps and o/ps
3. Reverse the direction of flow of signals in the direct form II structure.
4. Interchange nodes and summing points



iv) Cascade Realization Technique:

Cascade Realization is implemented by attaching all the factors of system function H(z) is in series that is cascade

Let us consider the ~~Nth~~ Nth order system is described by system function

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

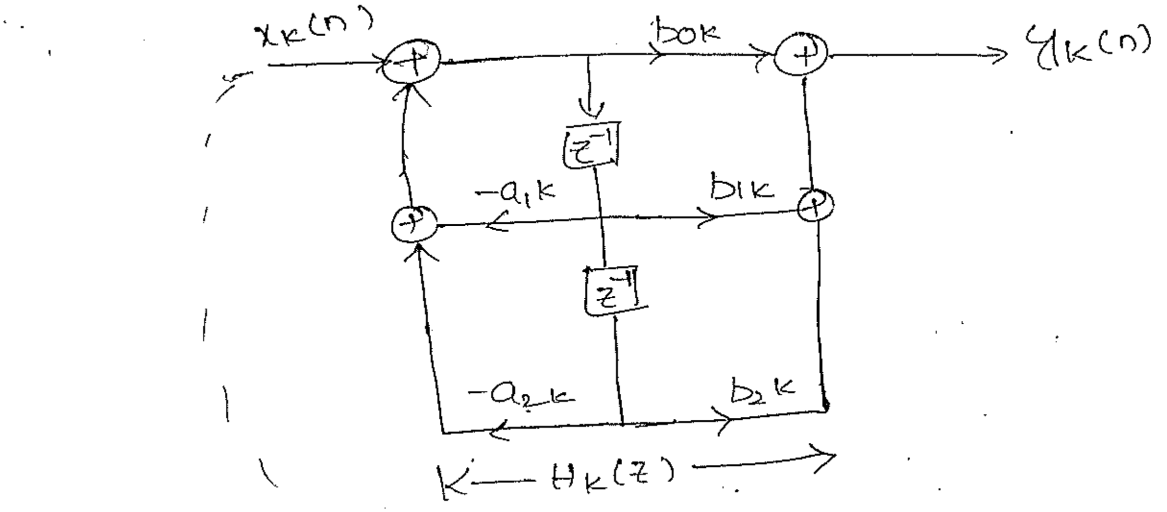
Let the system function H(z) is represented in [illegible] subsystems, each ~~high~~ subsystem's highest degree is '2'.

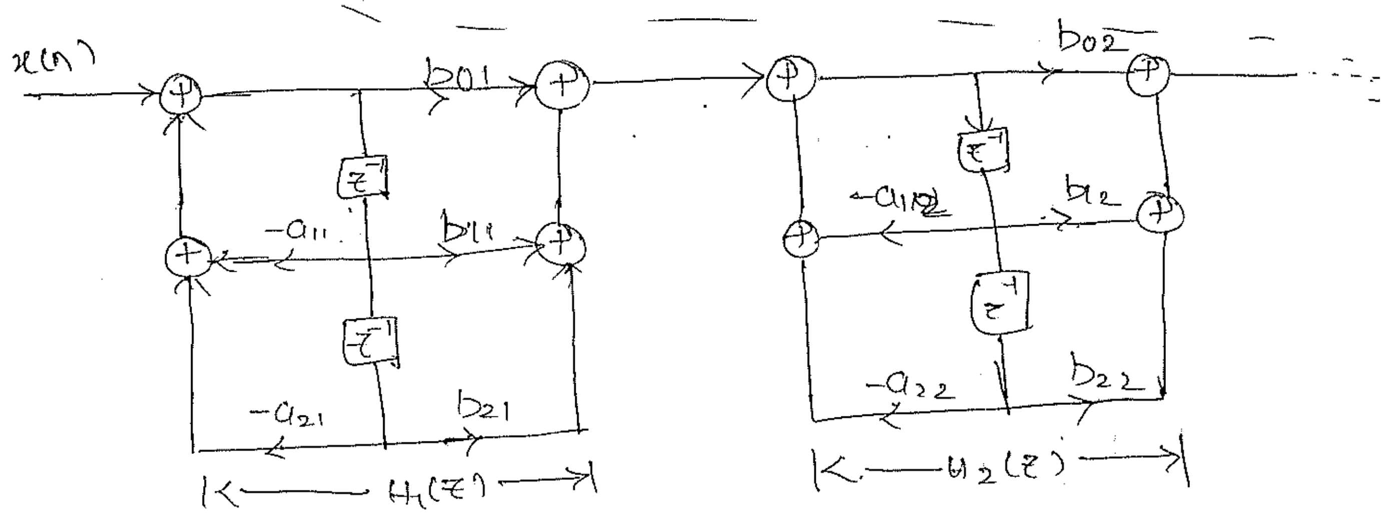
i.e., $H(z) = \prod_{k=1}^{N} H_k(z) = H_1(z) \cdot H_2(z) --- H_N(z)$

Here, N - Integer part of $\left(\frac{N+1}{2}\right)$

π - product of subsystem of H(z)

$$H_k(z) = \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}}$$





V) parallel form Realization technique:

parallel form realization is obtained by making use of partial fraction expansion of system function H(z).

let us consider N^{th} order system which is described by H(z).

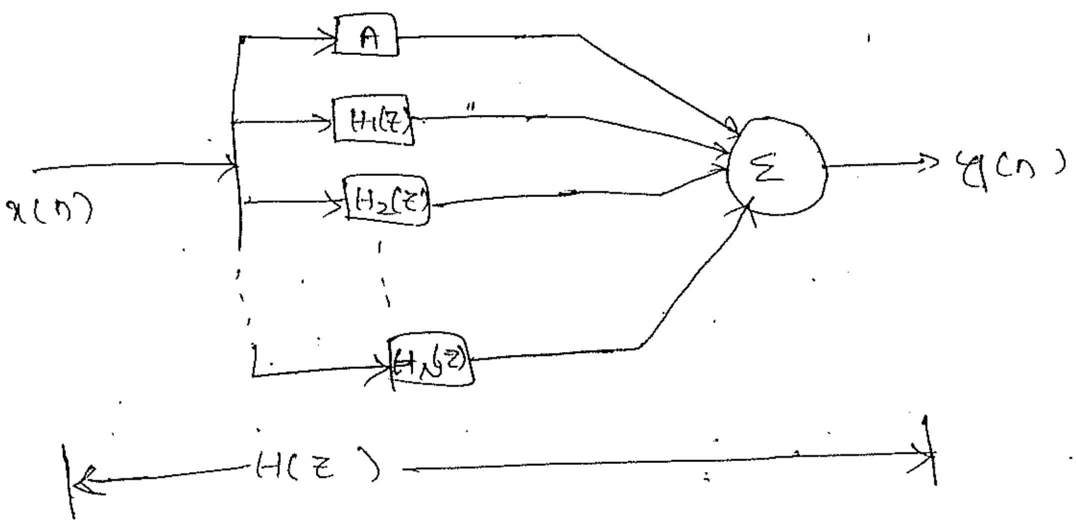
i.e.,

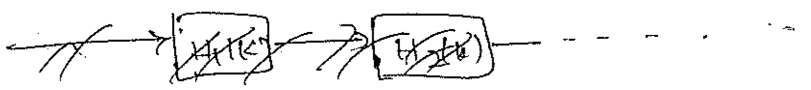
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{K=0}^{m} b_K z^{-K}}{1+\sum_{K=1}^{N} a_K z^{-K}} = A + \sum_{K=1}^{N} \frac{A_K}{1-a_K z^{-1}}.$$

$$= A + \sum_{K=1}^{N} H_K(z).$$

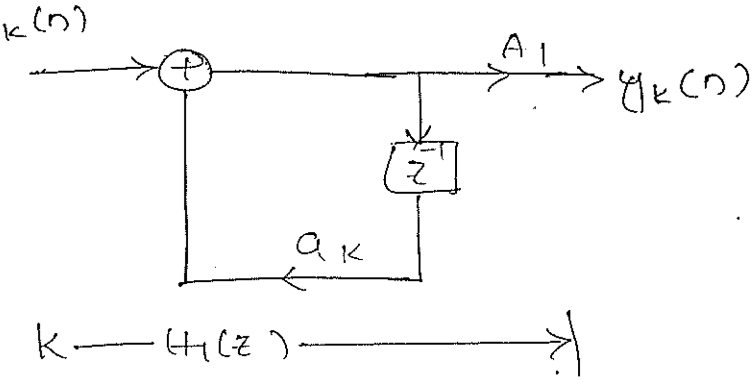
$$H(z) = A + \sum_{k=1}^{N} H_K(z) = A + H_1(z) + H_2(z) + \text{----} + H_N(z)$$

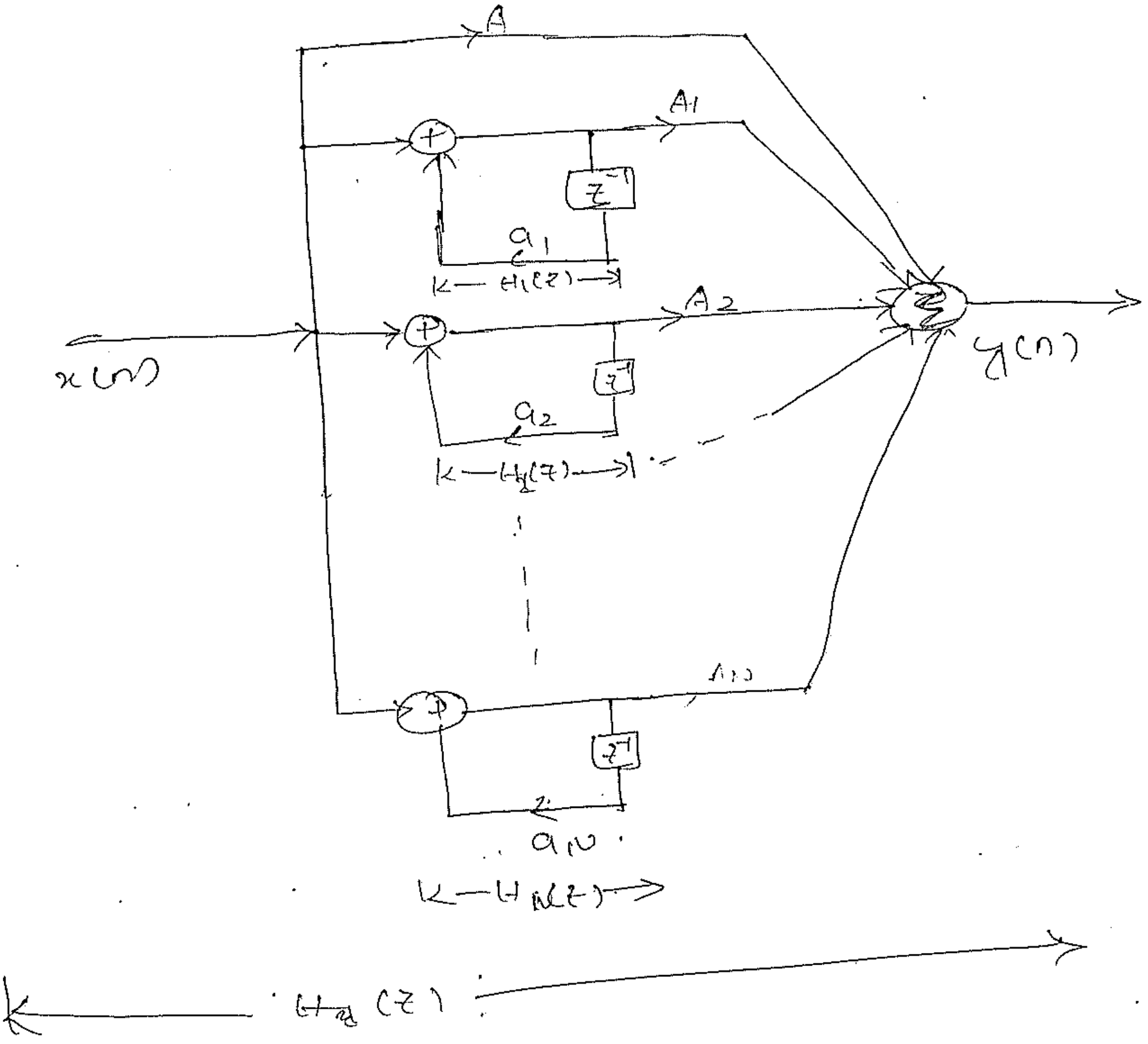
Here, $A = \frac{b_m}{a_N}$





$$H_K(z) = \frac{A_K}{1-a_K z^{-1}}$$





Problems:

1. Realize the following systems.

i. $y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$. by using

i) Direct form - I, Direct form II & transposed Structure.

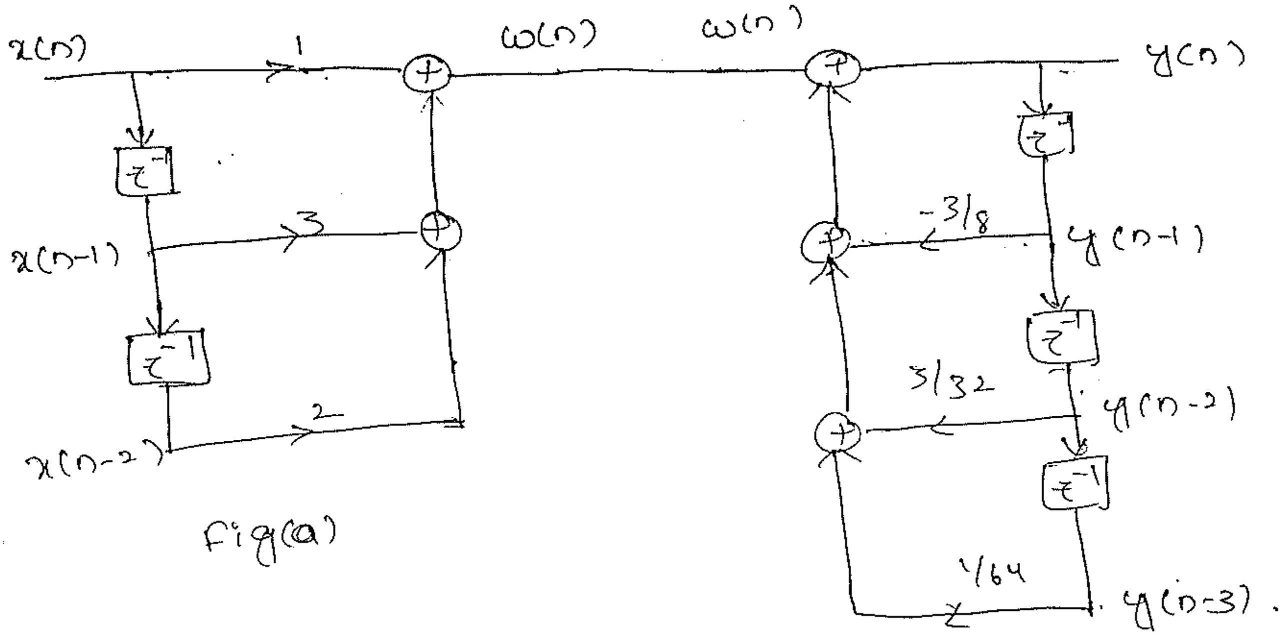
i) Direct form I

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2).$$

Let $w(n) = x(n) + 3x(n-1) + 2x(n-2)$, then. $\longleftarrow$ ①

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + w(n) \longleftarrow \text{(ii)}.$$

Eqn can be realized as show in figa.
& eqn ② as shown in fig(b)



Fig(a)

Fig(b).

fig(c).

This realization requires

Total no. of multipliers $= M+N+1$

$= 2+3+1$

$= 6.$

Summers $= M+N$

$= 2+3 = 5$

Memory location $= M+N$

$= 2+3$

$= 5.$

Direct form II :-

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n)$$

$$+ 3x(n-1) + 2x(n-2)$$

Applying Z.T on both sides, we get

$$Y(z) = -\frac{3}{8}z^{-1}Y(z) + \frac{3}{32}z^{-2}Y(z) + \frac{1}{64}z^{-3}Y(z) + X(z)$$

$$+ 3z^{-1}X(z) + 2z^{-2}X(z).$$

$$Y(z)\left[1 + 3/8\,z^{-1} - 3/32\,z^{-2} - 1/64\,z^{-3}\right] = X(z)\left[1 + 3z^{-1} + 2z^{-2}\right].$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + 3/8\,z^{-1} - 3/32\,z^{-2} - \frac{1}{64}z^{-3}}$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + 3/8\,z^{-1} - 3/32\,z^{-2} - \frac{1}{64}z^{-3}}$$

$$\omega(z)\left[1+3/8\, z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}\right] = X(z).$$

$$\omega(z) = X(z) - 3/8\, z^{-1}\omega(z) + \frac{3}{32} z^{-2}\omega(z) + \frac{1}{64} z^{-3}\omega(z)$$

Applying Inverse z - Transform.

$$\omega(n) = x(n) - 3/8\,\omega(n-1) + \frac{3}{32}\omega(n-2) + \frac{1}{64}\omega(n-3) \quad \text{—— ①}.$$

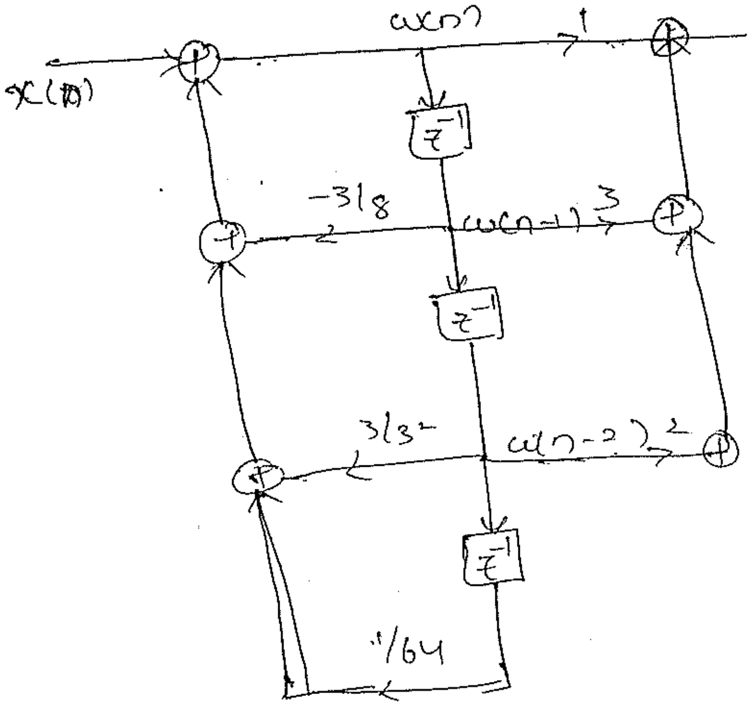
$$\frac{Y(z)}{\omega(z)} = 1+3z^{-1}+2z^{-2}$$

$$Y(z) = \left[1+3z^{-1}+2z^{-2}\right]\omega(z)$$

$$= \omega(z) + 3z^{-1}\omega(z) + 2z^{-2}\omega(z).$$

Apply IZT.

$$y(n) = \omega(n) + 3\omega(n-1) + 2\omega(n-2) \quad \text{—— ②}$$



multipliers = M+N+1

= 2+3+1

= 6.

adders = M+N

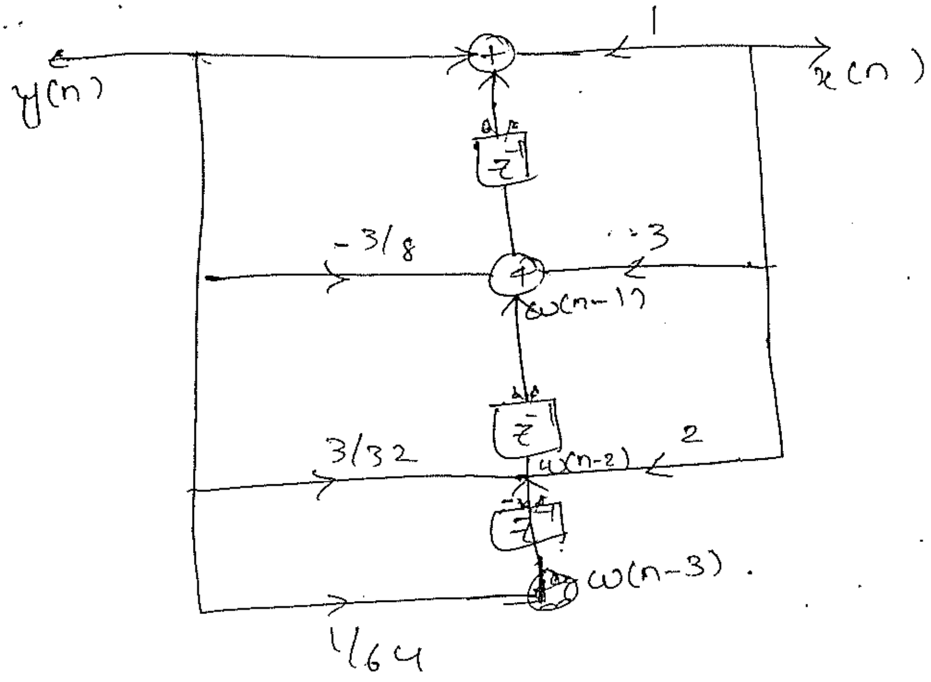
= 2+3

= 5.

Memory = max[2, 3]

= 3.

Transposed structure:



multipliers =

2. Realise the following system using Direct form I, Direct form II & transposed structure.

$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{(z - 1/4)(z^2 - z + 1/2)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-3}\left[8z^3 - 4z^2 + 11z - 2\right]}{z^{-3}\left[(z - 1/4)(z^2 - z + 1/2)\right]}$$

$$= \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - z^{-1} + \frac{z^{-2}}{2} - \frac{1}{4}z^{-1} + \frac{z^{-2}}{4} - \frac{1}{8}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4\,z^{-1} + 3/4\,z^{-2} - \frac{1}{8}z^{-3}}.$$

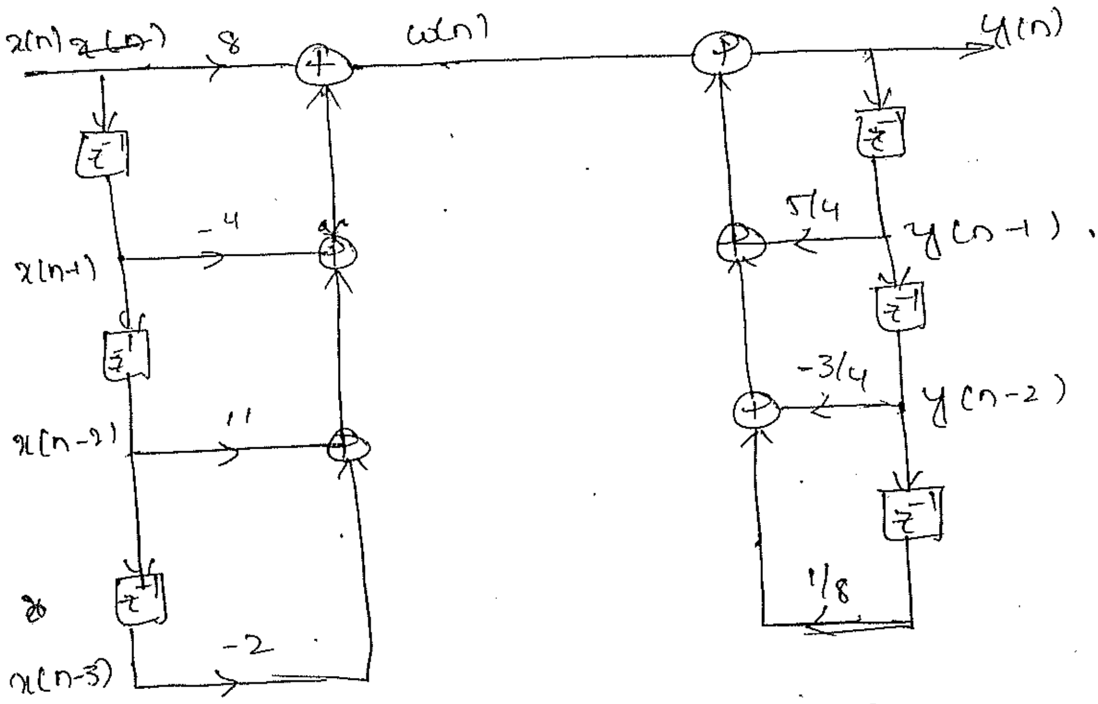
$$Y(z)\,\cancel{H(z)}\left[1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}\right]=\left(8-4z^{-1}+11z^{-2}-2z^{-3}\right)X(z)$$

Apply I-Z-T.

$$y(n)=\frac{5}{4}y(n-1)-\frac{3}{4}y(n-2)+\frac{1}{8}y(n-3)+8x(n)-4x(n-1)+11x(n-2)-2x(n-3)$$

$$w(n)=8x(n)-4x(n-1)+11x(n-2)-2x(n-3).$$

$$y(n)=\frac{5}{4}y(n-1)-\frac{3}{4}y(n-2)+\frac{1}{8}y(n-3)+w(n).$$



Direct form I.

adders = M+N+0

= 3+3+0 = 6.

multipliers = M+N+1 = 7.

Memory = M+N

= 6.

ii)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4\, z^{-1} + 3/4\, z^{-2} - \frac{1}{8} z^{-3}}$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} .$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - 5/4\, z^{-1} + \frac{3}{4} z^{-2} - \frac{1}{8} z^{-3}}$$

$$W(z)\left[1 - 5/4\, z^{-1} + \frac{3}{4} z^{-2} \cancel{+} - \frac{1}{8} z^{-3}\right] = X(z) .$$

Apply I·Z·T

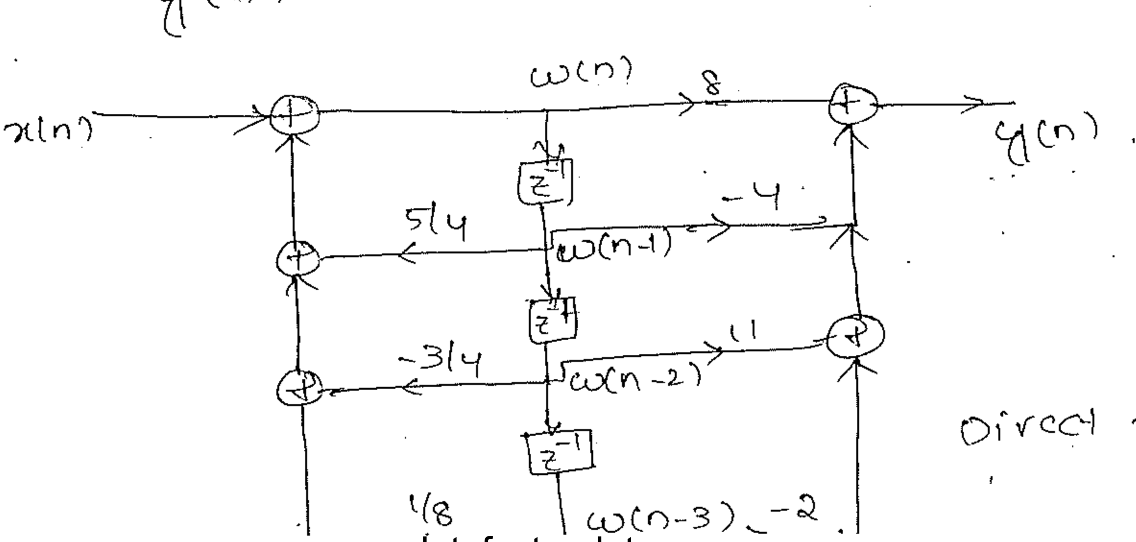
$$w(n) = x(n) + 5/4\, w(n-1) \mp 3/4\, w(n-2) + \frac{1}{8} w(n-3)$$

$$\frac{Y(z)}{W(z)} = 8 - 4z^{-1} + 11z^{-2} - 2z^{-3} .$$

$$Y(z) = (8 - 4z^{-1} + 11z^{-2} - 2z^{-3})\, W(z) .$$

Apply I·Z·T .

$$y(n) = 8w(n) - 4w(n-1) + 11w(n-2) - 2w(n-3$$



Direct form II

IV

multipliers = M+N+1

= 3+3+1 = 7.

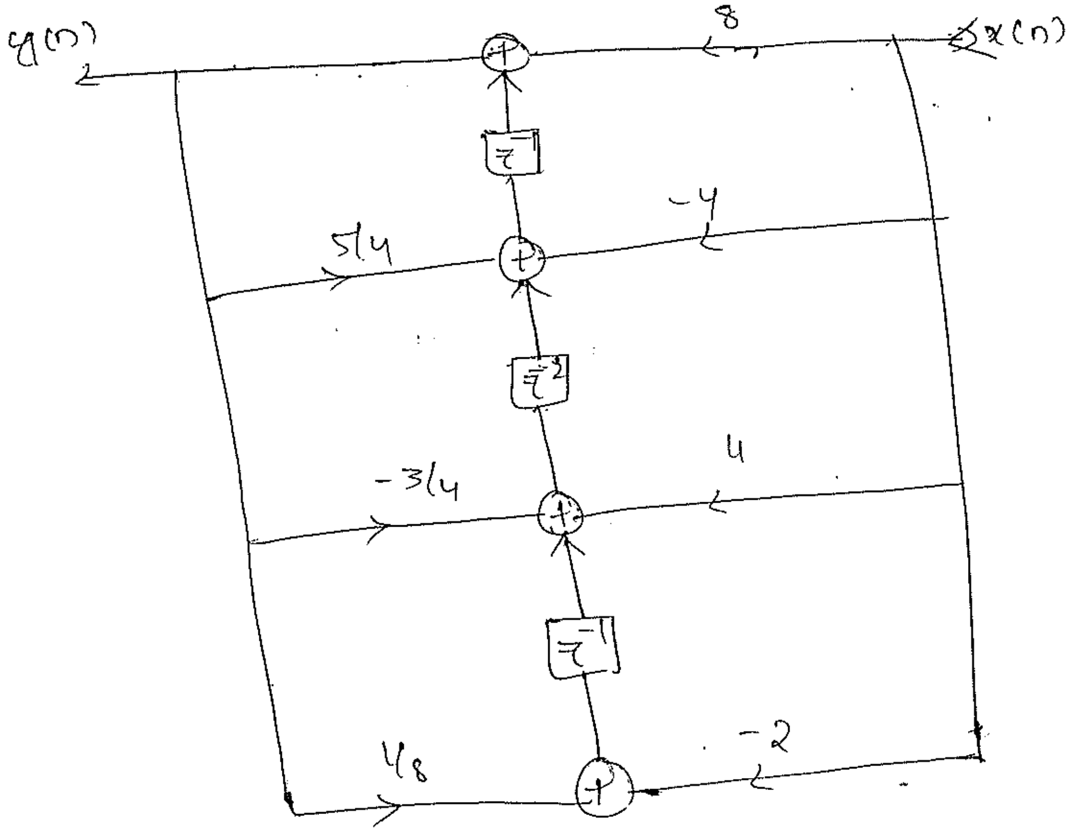
Adders = M+N

= 3+3 = 6.

Memory = max [M, N]

= 3.

Transposed structure:



* Realize the following system.

$y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + 3x(n) + 3.6\, x(n-1) + 0.6\, x(n-2)$. by using i) Direct form I ii) Direct form II iii) ~~Cascas~~ Transposed iv) cascade v) parallel form realizations.

Sol:-

Cascade Realization:

$y(n) = -0.1 y(n-1) + 0.2 y(n-2) + 3x(n) + 3.6x(n-1)$

$+ 0.6\, x(n-2)$.

Applying z-transform on both sides

$$Y(z) = -0.1 z^{-1} Y(z) + 0.2 z^{-2} Y(z) + 3X(z) + 3.6 z^{-1} X(z) + 0.6 z^{-2} X(z).$$

$$Y(z)\left[1+0.1z^{-1}-0.2z^{-2}\right] = \left[3+3.6z^{-1}+0.6z^{-2}\right]X(z).$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}} \times \frac{z^2}{z^2}.$$

$$= \frac{3z^2+3.6z+0.6}{z^2+0.1z-0.2}$$

$$= \frac{(3z+0.6)(z+1)}{(z+0.5)(z-0.4)}.$$

$$H(z) = \left[\frac{3z+0.6}{z+0.5}\right]\left[\frac{z+1}{z-0.4}\right] = \left[\frac{3+0.6z^{-1}}{1+0.5z^{-1}}\right]\left[\frac{1+z^{-1}}{1-0.4z^{-1}}\right]$$

$$= H_1(z) \cdot H_2(z).$$

$$H_k(z) = \frac{\cancel{3+0.6z^{-1}}}{\cancel{1+0.5z^{-1}}} \cdot \frac{A_k+b_k z^{-1}}{1+a_k z^{-1}} = \frac{Y_k(z)}{X_k(z)} = \frac{Y_k(z)}{W_k(z)} \cdot \frac{W_k(z)}{X_k(z)}$$

$$\frac{W_k(z)}{X_k(z)} = \frac{1}{1+a_k z^{-1}} \Rightarrow W_k(z) + W_k(z)\, a_k z^{-1} = X_k(z).$$

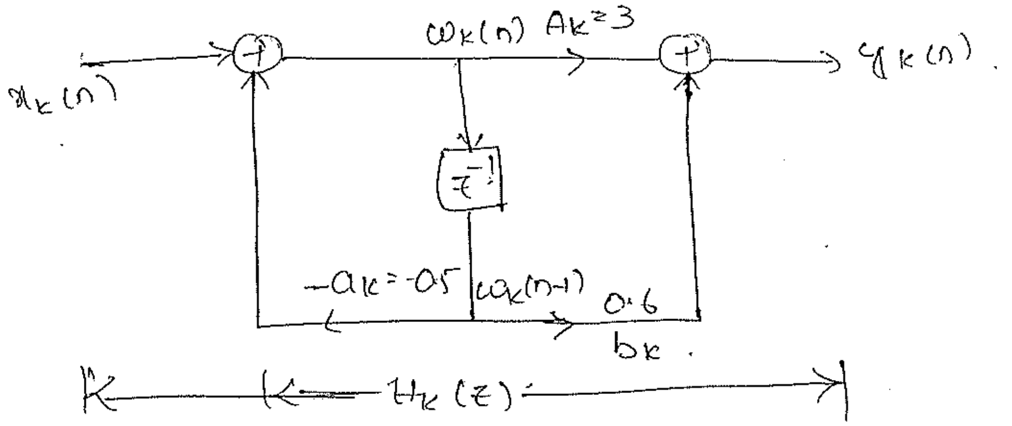
$$W_k(z) = X_k(z) - a_k z^{-1} W_k(z).$$

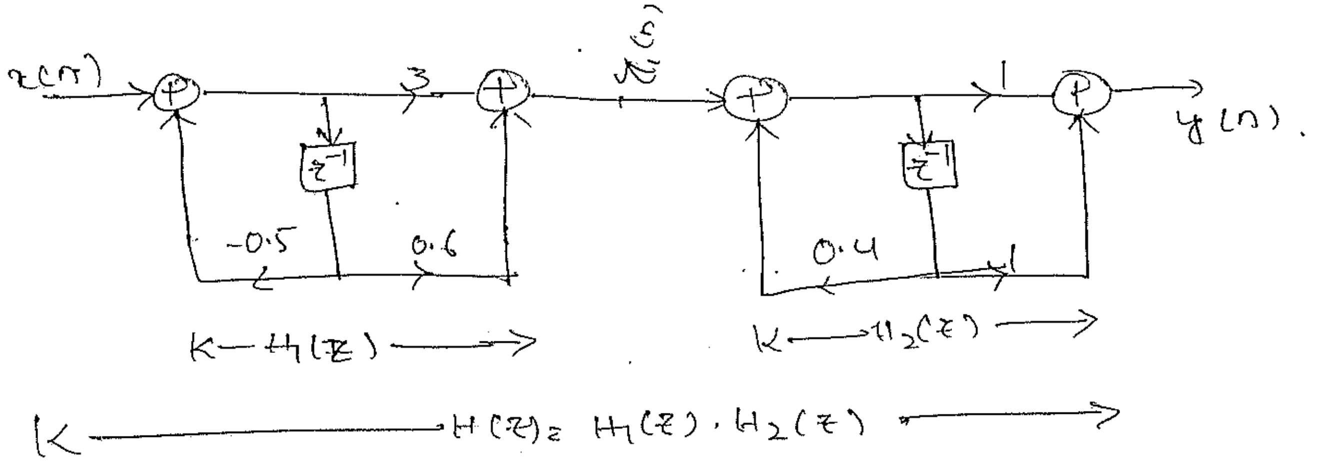
$$w_k(n) = x_k(n) - a_k w_k(n-1) \longrightarrow ①.$$

$$\frac{Y_k(z)}{W_k(z)} = A_k + b_k z^{-1} \Rightarrow Y_k(z) = A_k W_k(z) + b_k z^{-1} W_k(z)$$

$$y_k(n) = A_k w_k(n) + b_k w_k(n-1)$$

$$\longleftarrow ②.$$





Parallel form Realization:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= \frac{3z^2+3.6z+0.6}{z^2+0.1z-0.2} = \frac{(3z+0.6)(z+1)}{(z+0.5)(z-0.4)}$$

$$= 3 + \frac{3.3z+1.2}{z^2+0.1z-0.2}$$

$$z^2+0.1z-0.2 \overline{\big) 3z^2+3.6z+0.} \quad 3$$
$$3z^2+0.3z-0.$$
$$(-)\ (-)\ (+)$$
$$3.3z+1$$

$$= 3 + \frac{3.3z+1.2}{(z+0.5)(z-0.4)}$$

$$\frac{H(z)}{z} = \cancel{3z+0.6} \cdot \frac{3z^2+3.6z+0.6}{z(z^2+0.1z-0.2)} = \frac{3z^2+3.6z+0.6}{z(z+0.5)(z-0.4)}$$

$$= \frac{A}{z} + \frac{B}{z+0.5} + \frac{C}{z-0.4}$$

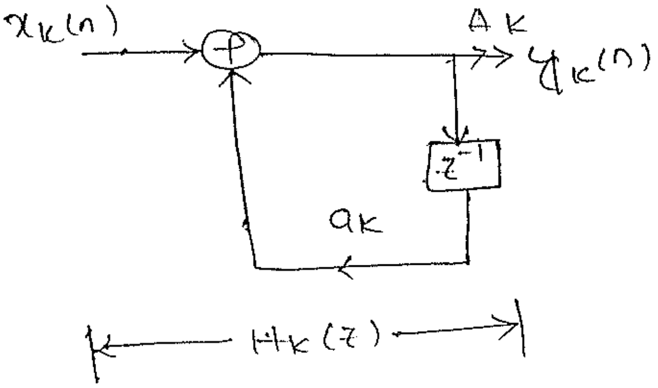
$$\frac{H(z)}{z} = \frac{-3}{z} + \frac{-1}{z+0.5} + \frac{7}{z-0.4}$$

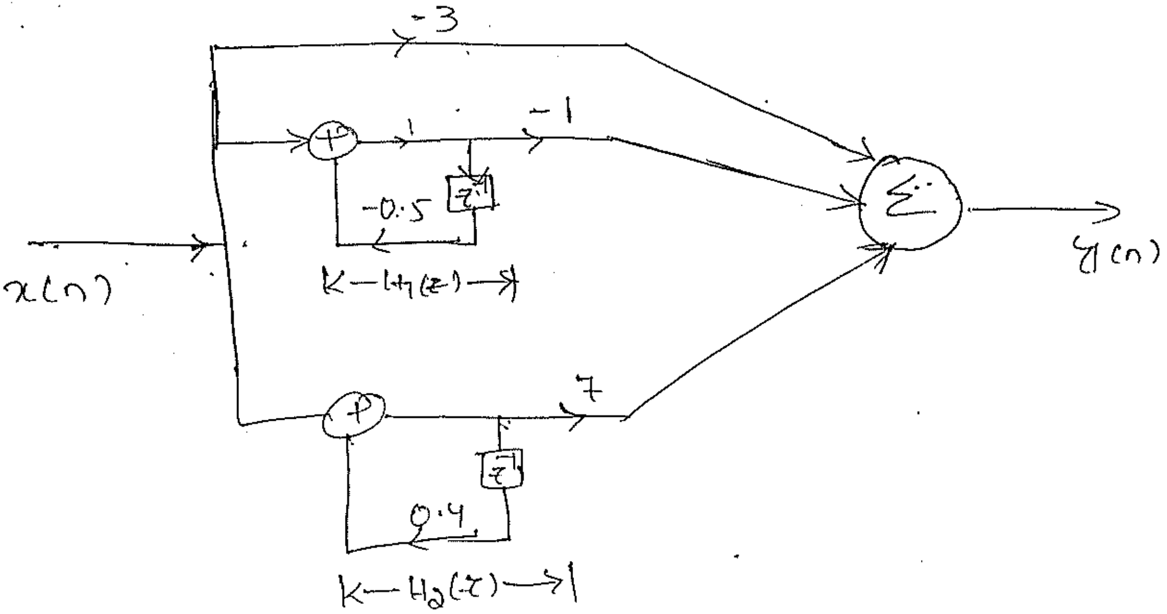
$$H(z) = -3 + \frac{-z}{z+0.5} + \frac{7z}{z-0.4}$$

$$H(z) = -3 + \frac{-1}{1+0.5z^{-1}} + \frac{7}{1-0.4z^{-1}}$$

$$= A + H_1(z) + H_2(z).$$

$$H_k(z) = \frac{A_k}{1-a_k z^{-1}} = \frac{y_k(z)}{x_k(z)}$$





FIR Digital Filters structures (or) Non Recursiv Realization techniques:

There are 3 well known techniques to realize FIR digital filters.

1. Direct form realization (or) Transversal realizati

2. -on (or) Tapped delay line filter.

2. cascade Realization

3. Linear phase Realization (or) Min no.of multiplier realization technique.

1. Direct form Realization technique:

If the system having finite duration impulse response ~~sys~~ sequence then it is called FIR system.

If the length of impulse response is N

i.e.,

$$h(n) = \{h(0), h(1), h(2), --- h(N-1)\};$$

length

$$T.F = H(z) = \frac{Y(z)}{X(z)} = Z.T\left[h(n)\right] = \sum_{n=0}^{N-1}\left[h(n)\right]z^{-n}$$

$$= \sum_{K=0}^{N-1}\left[h(k)\right]z^{-k}.$$

$$Y(z) = \sum_{K=0}^{N-1} h(K) z^{-K} X(z).$$

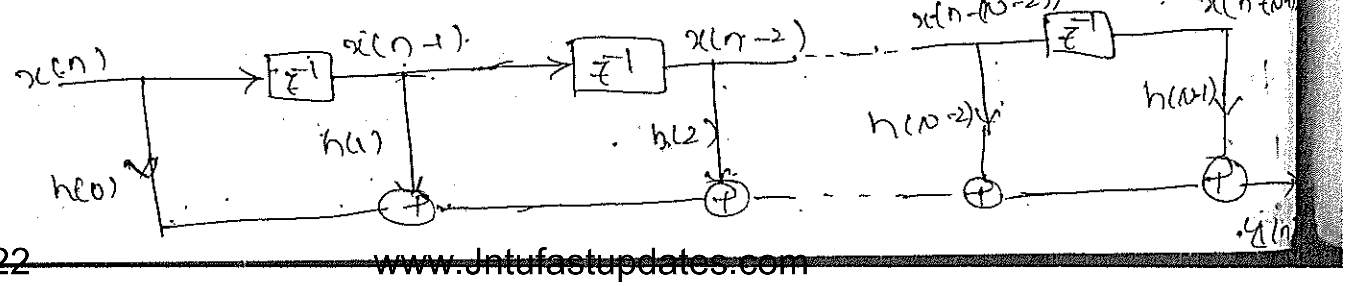
Applying I.Z.T & linearity property, we get.

$$I.Z.T\left[Y(z)\right] = I.Z.T\left[\sum_{K=0}^{N-1} h(K) z^{-K} \cdot X(z)\right]$$

$$y(n) = \sum_{K=0}^{N-1} h(k)\ I.Z.T\left[z^{-K} X(z)\right]$$

$$= \sum_{K=0}^{N-1} h(k)\ x(n-k).$$

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + --- + h(N-1)\,x(n-(N-1)) \longrightarrow ①.$$



This realization requires,

total no. of multipliers = N.

total no. of adders = N-1.

Memory locations = N-1.

Cascade Realization:

This realization is implemented all the subsystems of system function H(z) can be connected in series i.e., cascade.

Case(i): If 'N' is odd

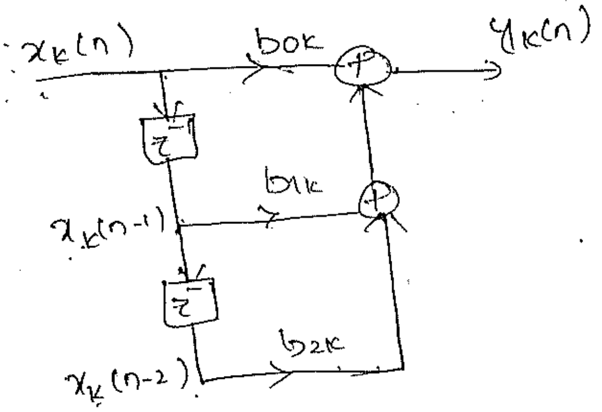
$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} H_k(z) = \prod_{k=1}^{\frac{N-1}{2}} \left[b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}\right]$$

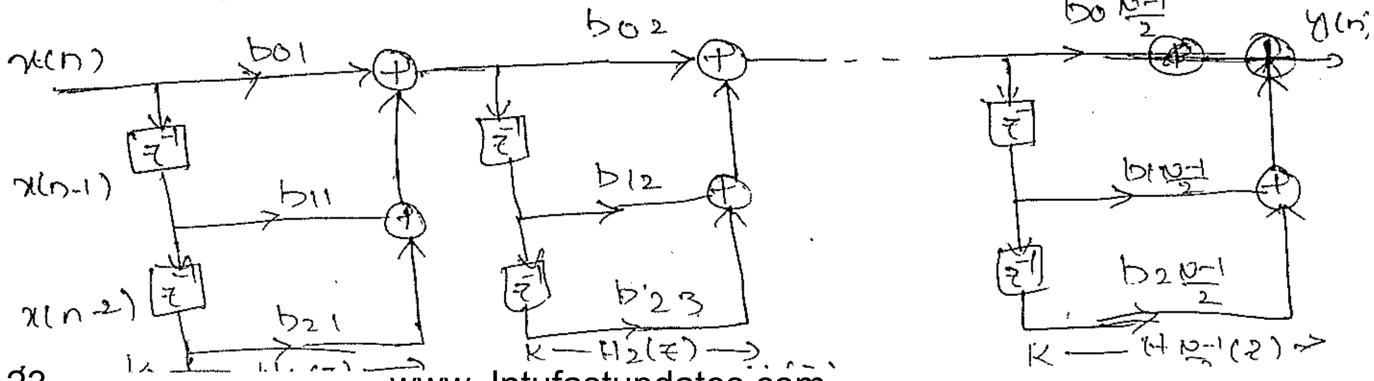
$$= H_1(z)\cdot H_2(z)\cdots H_{\frac{N-1}{2}}(z)$$

N = odd, $\frac{N-1}{2}$ = Even & H(z) have $\left(\frac{N-1}{2}\right)$ second order systems.

$$H_k(z) = \frac{Y_k(z)}{X_k(z)} = b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}$$

$$y_k(n) = b_{0k}x_k(n) + b_{1k}x_k(n-1) + b_{2k}x_k(n-2) \quad \text{—①}$$





If N is even, it has one first order system and ~~$\frac{N-1}{2}$~~ $\frac{N}{2}-1$ second order systems.

$$H(z) = (b_{01}+b_{11}z^{-1}) \cdot \prod_{k=2}^{N/2} H_k(z)$$

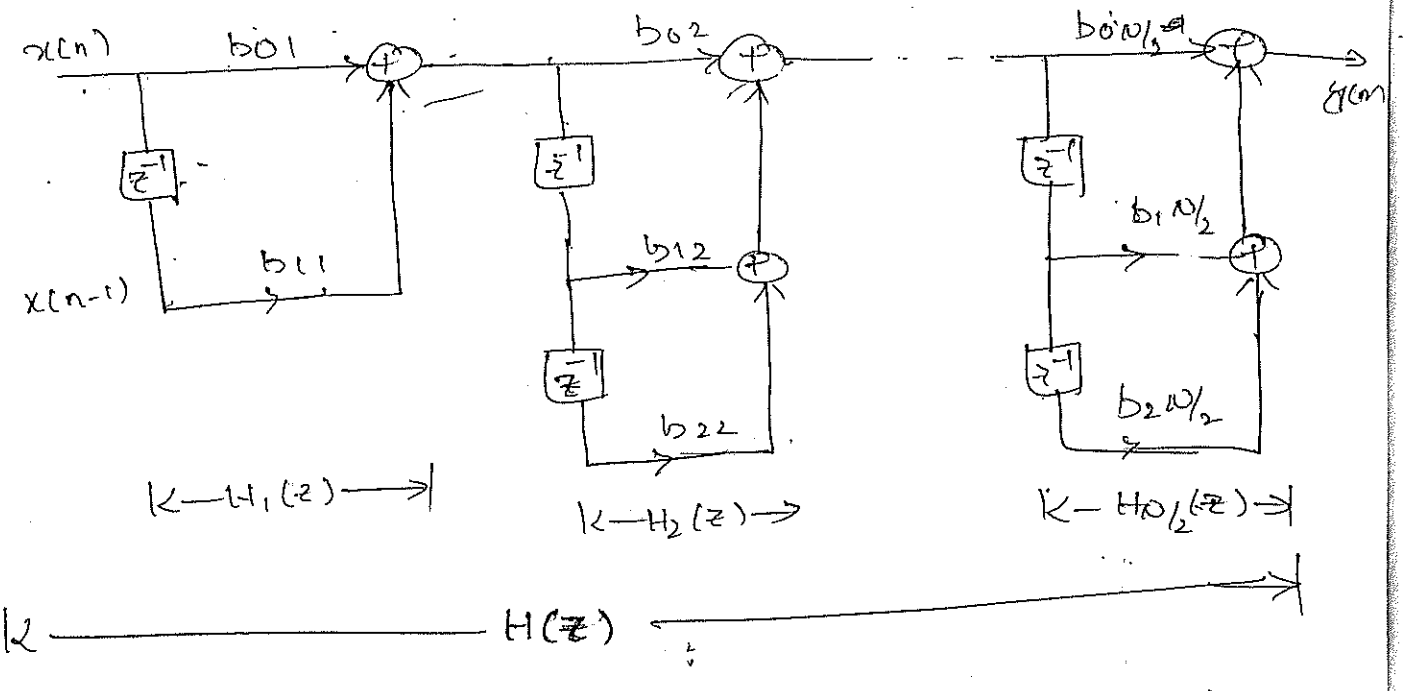
$$= (b_{01}+b_{11}z^{-1}) \prod_{k=2}^{N/2} \left[b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}\right]$$

$$= (b_{01}+b_{11}z^{-1}) H_2(z) \cdot H_3(z) \cdots H_{\frac{N}{2}}(z).$$

$$H_k(z) = \frac{Y_k(z)}{X_k(z)} = b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}$$

$$\cancel{H_k(z)} = \frac{Y_k(z)}{\cancel{X_k(z)}} = b$$

$$y_k(n) = b_{0k}x_k(n)+b_{1k}x_k(n-1) + b_{2k}x_k(n-2)$$



* Realize the following system using direct form realization.

$$h(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2) + 2\delta(n-4) + 5\delta(n-6) - \delta(n-7).$$

Sol:- $h(n) = \{\underset{0}{1}, \underset{1}{-2}, \underset{2}{3}, \underset{3}{0}, \underset{4}{2}, \underset{5}{0}, \underset{6}{5}, \underset{7}{-1}\}$

$$H(z) = Z.T[h(n)] = \sum_{n=-\infty}^{\infty} [h(n)] z^{-n}$$

em

$$= \sum_{n=0}^{7} h(n)z^{-n}$$

$$= h(0)+h(1)z^{-1}+h(2)z^{-2}+h(3)z^{-3}+h(4)z^{-4}+ h(5)z^{-5}+h(6)z^{-6}+h(7)z^{-7}.$$

$$H(z) = \frac{Y(z)}{X(z)} = 1-2z^{-1}+3z^{-2}+2z^{-4}+5z^{-6}-z^{-7}.$$

$$Y(z) = X(z) - 2z^{-1}X(z).$$

$$Y(z) = X(z)-2z^{-1}X(z)+3z^{-2}X(z)+2z^{-4}X(z)+ 5z^{-6}X(z)-z^{-7}X(z).$$

$_{2k}z^{-2}$

)

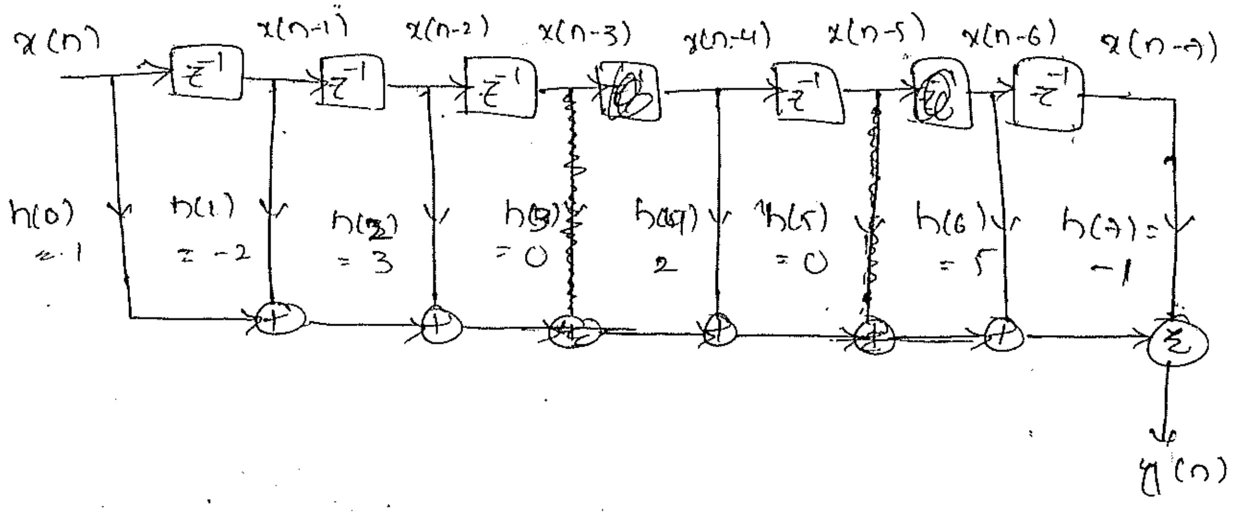
-2)

→ y(n)

Apply I.Z.T & linearity property of Z.T

we get.

$$y(n) = x(n) - 2x(n-1)+3x(n-2)+2x(n-4)+5 x(n-6) - x(n-7) \text{ —— ①}.$$



6) * Realize the following system using
1. Cascade Realization
2. Direct form Realization.

$$H(z) = (1 - z^{-1}+2z^{-2}+3z^{-3}-z^{-4})(2-z^{-1}-4z^{-3}+5z^{-5})$$

$$H(z) = H_1(z) \cdot H_2(z).$$

$$H_1(z) = 1 - z^{-1}+2z^{-2}+3z^{-3}-z^{-4} = \frac{Y_1(z)}{X_1(z)}$$

$$H_2(z) = 2 - z^{-1} - 4z^{-3}+5z^{-5} = \frac{Y_2(z)}{\;}$$

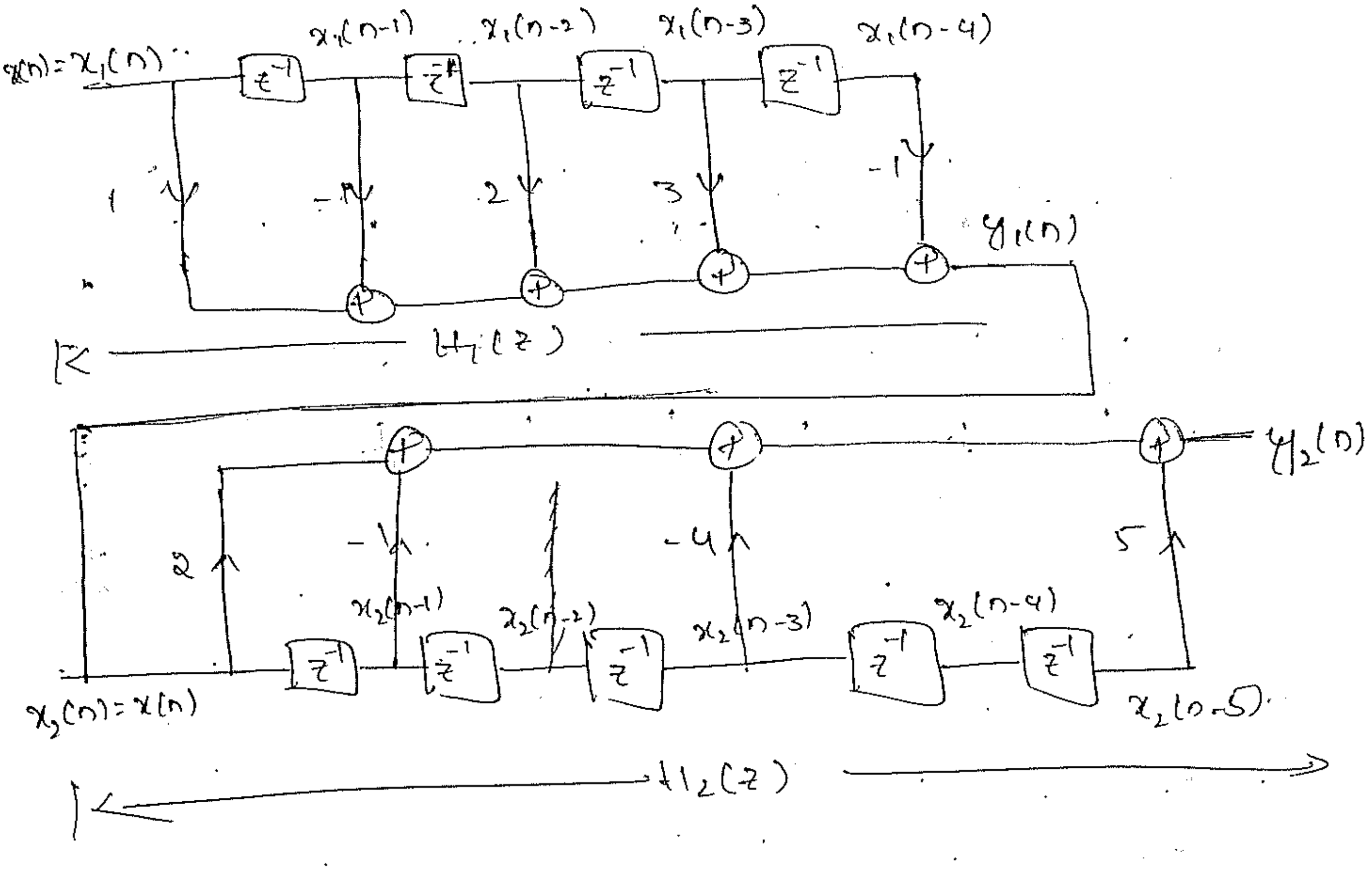
$y_1(n) =$

$$Y_1(z) = X_1(z) - z^{-1}X_1(z) + 2z^{-2}X_1(z) + 3z^{-3}X_1(z)$$
$$- X_1(z)z^{-4}.$$

$$y_1(n) = x_1(n) - x_1(n-1) + 2x_1(n-2) + 3x_1(n-3)$$
$$- x_1(n-4) \text{ —— } ①$$

$$Y_2(n) = 2X_2(z) - X_2(z)z^{-1} - 4z^{-3}X_2(z) + 5z^{-5}X_2(z)$$

$$y_2(n) = 2x_2(n) - x_2(n-1) - 4x_2(n-3) + 5x_2(n-5)$$
$$\text{—— } ②$$



Direct form:

$$H(z) = (1 - z^{-1} + 2z^{-2} + 3z^{-3} - z^{-4})(2 - z^{-1} - 4z^{-3} + 5z^{-5})$$
$$= 2 - z^{-1} - 4z^{-3} + 5z^{-5} - 2z^{-1} + z^{-2} + 4z^{-4} - 5z^{-6}$$
$$+ 4z^{-2} - 2z^{-3} - 8z^{-5} + 10z^{-7} + 6z^{-3} - 3z^{-4} -$$
$$12z^{-6} + 15z^{-8} - 2z^{-4} + z^{-5} + 4z^{-7} - 5z^{-9}$$

$= 2 - 3z^{-1} + 5z^{-2} + z^{-4}$

$_1(z)$

$)-3)$

$x_2(z)$

$n-5)$

$n)$

$z^{-5})$

-6

Linear phase Realization technique/ Minimum no.of multipliers required realization:-

for linear phase FIR digital filter, impulse response satisfies the following condition.

$$h(n) = h(N-1-n) \ \forall \ 'n'.$$

N - length of h(n)

$$h(n) = \{ h(0), h(1), h(2), ---- h(N-1) \}.$$

$$T.f = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + ---$$
$$--- + h(N-1)z^{-(N-1)}.$$

Case(i) :-

If 'N' is even

$$T.F = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{N-1} [h(n)] z^{-n} + \sum_{n=\frac{N}{2}}^{N-1} [h(n)] z^{-n}$$

put $n = N-1-m$.

$m = N-1-n$.

$n = N/2$; $m \rightarrow (N-1) - \frac{N}{2} = \frac{N}{2} - 1$.

$n = N-1$; $m \rightarrow (N-1)-(N-1) = 0$.

$$N/ \quad z^{-n} + \sum_{m=\frac{N}{2}-1}^{0} h(N-1-m) z^{-(N-1-m}$$

put m = n.

$$= \sum_{n=0}^{N/2-1} h(n) z^{-n} + \sum_{n=0}^{N/2-1} h(N-1-n) z^{-(N-1-n}$$

$\because h(n) = h(N-1-n)$.

$$= \sum_{n=0}^{N/2-1} h(n) z^{-n} + \sum_{n=0}^{N/2-1} h(n) z^{-(N-1-n)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N/2-1} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

$$Y(z) = \sum_{n=0}^{N/2-1} h(n) \left[z^{-n} X(z) + z^{-(N-1-n)} X(z) \right]$$

put n = k

$$Y(z) = \sum_{k=0}^{N/2-1} h(k) \left[z^{-k} X(z) + z^{-(N-1-k)} X(z) \right]$$

Applying I.Z.T & linearity property.

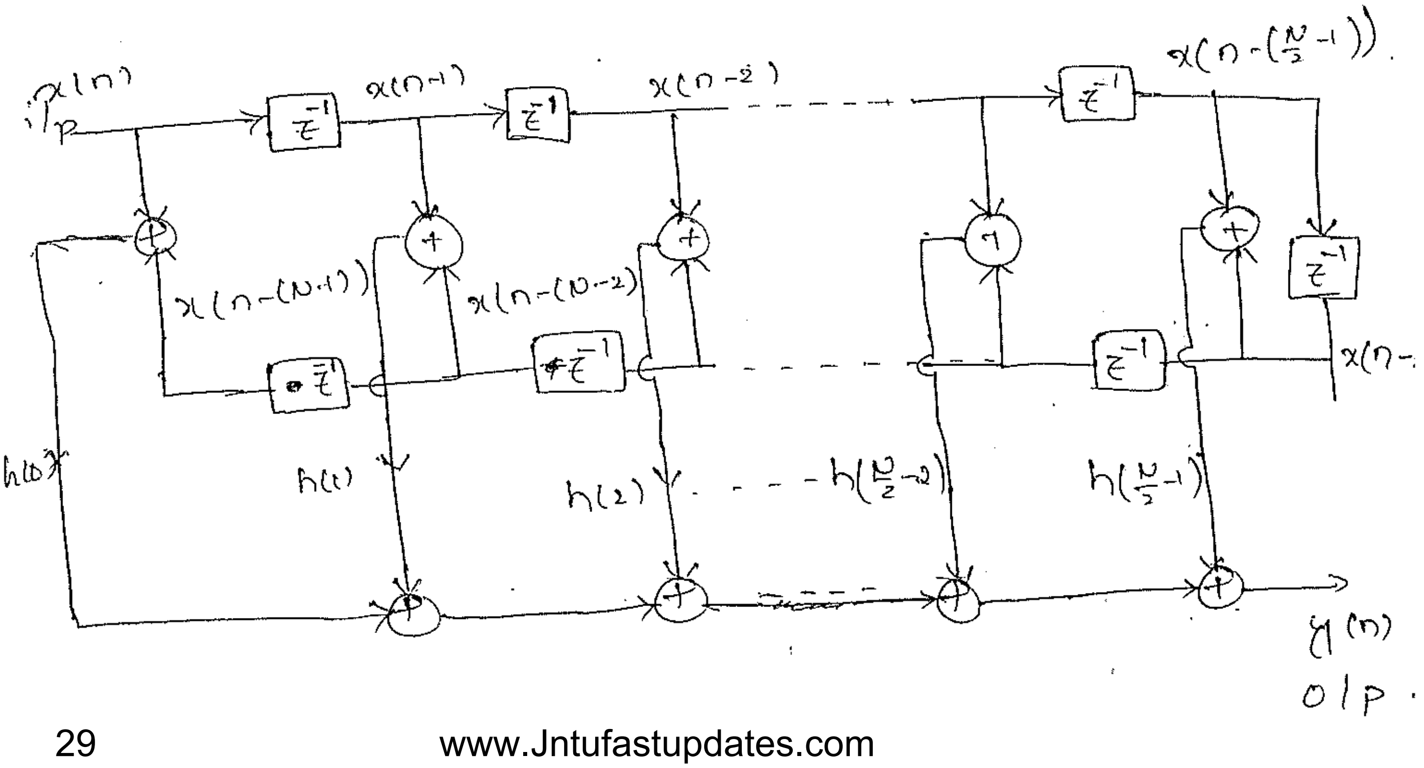
$$x(n-k) \longleftrightarrow z^{-k} X(z)$$

~~$y(n) =$~~

$$y(n) = \sum_{k=0}^{N/2-1} h(k) \left[x(n-k) + x(n-(N-1-k)) \right]$$

$$y(n) = h(0)\left[x(n) + x(n-(N-1)) \right] + h(1) \left[x(n-1) + x(n-(N-2)) \right] +$$

$$- - - - + h\left(\frac{N}{2}-1\right) \left[x\left(n-\left(N/2-1\right)\right) + x\left(n-\left(N/2\right)\right) \right] \quad —①$$



for Direct form realization, total no.of multipliers required = N.

for Linear phase realization & N is even, total no.of multipliers required = N/2.

Case(ii):-

If N is odd number.

$$T.F = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}.$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) z^{-n}.$$

put $n = N-1-m$

$m = N-1-n$.

$n \to \frac{N+1}{2}$; $m \to (N-1) - \left(\frac{N+1}{2}\right) = \frac{N-3}{2}$,

$n \to N-1$; $m = 0$.

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{m=\frac{N-3}{2}}^{0} h(N-1-m) z^{-(N-1-m)}$$

put $m = n$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) z^{-(N-1-n)}$$

$\because h(n) = h(N-1-n)$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-(N-1-n)}$$

~~$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}$~~

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

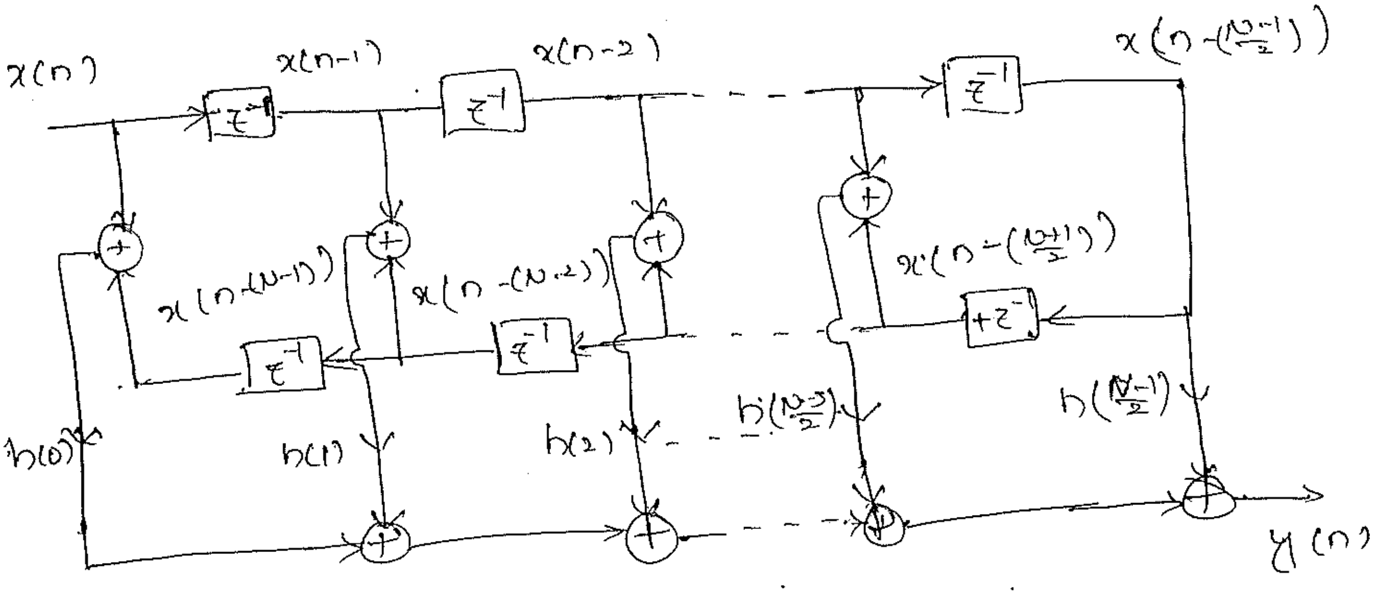
$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[z^{-k} + z^{-(N-1-k)}\right] + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

$$Y(z) = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[z^{-k}X(z) + z^{-(N-1-k)}X(z)\right] + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} X(z).$$

Applying I.Z.T & linearity prop.

$$y(n) = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[x(n-k) + x(n-(N-1-k))\right] + h\left(\frac{N-1}{2}\right) x\left(n-\left(\frac{N-1}{2}\right)\right)$$

$$y(n) = h(0)\left[x(n) + x(n-(N-1))\right] + h(1)\left[x(n-1) + x(n-(N-2))\right] + \cdots + h\left(\frac{N-3}{2}\right)\left[x\left(n-\left(\frac{N-3}{2}\right)\right) + x\left(n-\left(\frac{N+1}{2}\right)\right)\right] + h\left(\frac{N-1}{2}\right) x\left(n-\left(\frac{N-1}{2}\right)\right) \quad ①$$



for N = odd.

no.of multipliers $= \frac{N+1}{2}$.

* Realise the following system using..

i) Direct form ii) linear phase iii) ~~cascade R.T~~.

$$h(n) = \frac{1}{2}\delta(n) + \frac{1}{3}\delta(n-1) + \delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4) + \frac{1}{3}\delta(n-5) + \frac{1}{2}\delta(n-6).$$

Sol :- i) Direct form :-

$h(n) = \left\{ \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, 1/3, 1/2 \right\}$

length of $h(n) = N = 7$.

$$H(z) = Z.T\left[h(n)\right] = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{6} h(n) z^{-n}$$

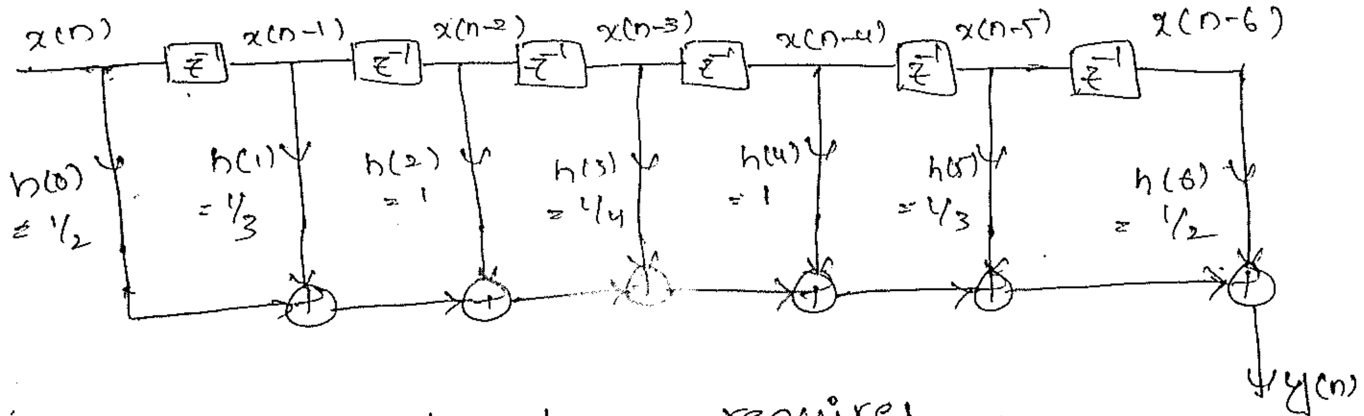
$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + 1/3\, z^{-5} + 1/2\, z^{-6}.$$

$$Y(z) = \frac{1}{2}X(z) + \frac{1}{3}z^{-1}X(z) + z^{-2}X(z) + \frac{1}{4}z^{-3}X(z) + z^{-4}X(z) + \frac{1}{3}z^{-5}X(z) + \frac{1}{2}z^{-6}X(z).$$

Applying I.Z.T & linearity prop.

$$\therefore y(n) = \frac{1}{2}x(n) + \frac{1}{3}x(n-1) + x(n-2) + \frac{1}{4}x(n-3) + x(n-4) + \frac{1}{3}x(n-5) + \frac{1}{2}x(n-6) \quad \text{——} ①,$$



This realization requires,

no.of multipliers = 7.

no.of adders = 7 - 1 = 6.

no.of memory ~~locations~~ elements = 7 - 1 = 6.

ii) <u>linear phase:</u>

we know condition for linear phase fil filter,

$h(n) = h(N-1-n), \forall n.$

$N = 7; \quad h(n) = h(6-n).$

$n = 0; \quad h(0) = h(6) = 1/2.$

$n = 1; \quad h(1) = h(5) = 1/3.$

$n = 2; \quad h(2) = h(4) = 1.$

$n = 3; \quad h(3) = 1/4 = h(3).$

∴ Given system satisfies linear phase condition.

$H(z) = z.T[h(n)] = \sum_{n=0}^{6} h(n) z^{-n}.$

$H(z) = \frac{Y(z)}{X(z)} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$

$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(2)z^{-4} + h(1)z^{-5} +$

$h(0)z^{-6}$.

$h(0)[1+z^{-6}]$

$= ~~h(0)~~ + h(1)[z^{-1}+z^{-5}] + h(2)[z^{-2}+z^{-4}] + h(3)z^{-3}$

$y(z) = h(0)[x(z) + z^{-6}x(z)] + h(1)[z^{-1}x(z) + z^{-5}x(z)] +$

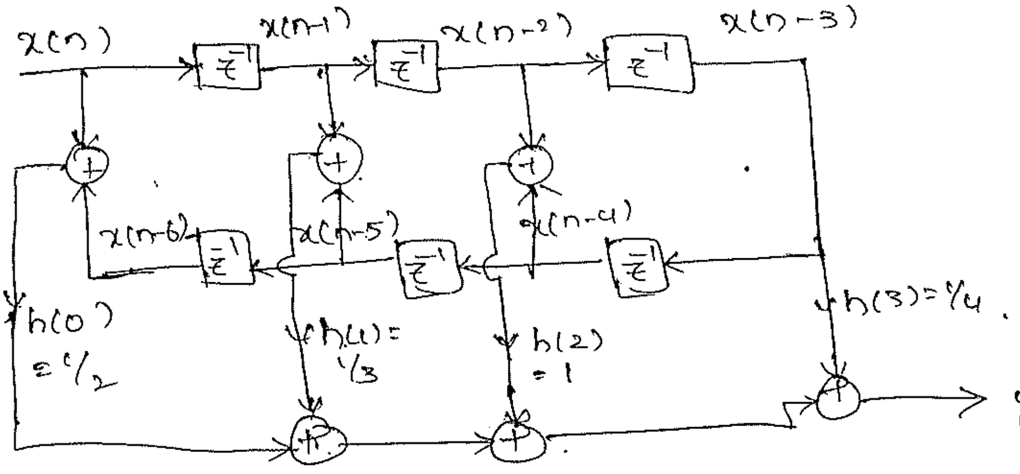
$h(2)[z^{-2}x(z) + z^{-4}x(z)] + h(3)z^{-3}x(z)$.

Applying I.Z.T & linearity prop.

$y(n) = h(0)[x(n) + x(n-6)] + h(1)[x(n-1) + x(n-5)]$

$+ h(2)[x(n-2) + x(n-4)] + h(3)x(n-3)$

——— ①



no. of multipliers $= \frac{N+1}{2} = 4$.

adders $= \frac{N-1}{2}$

$= 3$.

memory $= 6$.

2. $h(n) = \frac{1}{2}\delta(n) + \frac{1}{3}\delta(n-1) + \delta(n-2) + \frac{1}{4}\delta(n-3) +$

$\frac{1}{4}\delta(n-4) + \delta(n-5) + \frac{1}{3}\delta(n-6) + \frac{1}{2}\delta(n-7)$.

Sol:- Linear phase:

$h(n) = \{\underset{0}{\frac{1}{2}}, \underset{1}{\frac{1}{3}}, \underset{2}{1}, \underset{3}{1/4}, \underset{4}{1/4}, \underset{5}{1}, \underset{6}{1/3}, \underset{7}{1/2}\}$.

$N = 8$.

Condition for linear phase.

$h(n) = h(N-1-n) \ \forall \ 'n'$.

$n=0; \quad h(\cancel{7}-0) = h(0) = h(7) = 1/2$.

$n=1; \quad h(1) = h(6) = 1/3$.

$n=2; \quad h(2) = h(5) = 1$.

$n=3; \quad h(3) = h(4) = 1/4$.

∴ Given system satisfies linear phase condition.

$$H(z) = Z.T\left[h(n)\right] = \sum_{n=0}^{7} h(n) z^{-n}.$$

$$H(z) = \frac{Y(z)}{X(z)} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7}.$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(3)z^{-4} + h(2)z^{-5} + h(1)z^{-6} + h(0)z^{-}$$

$$= h(0)\left[1 + z^{-7}\right] + h(1)\left[z^{-1} + z^{-6}\right] + h(2)\left[z^{-2} + z^{-5}\right] + h(3)\left[z^{-3} + z^{-4}\right]$$

$$Y(z) = h(0)\left[X(z) + z^{-7}X(z)\right] + h(1)\left[z^{-1}X(z) + z^{-6}X(\right.$$

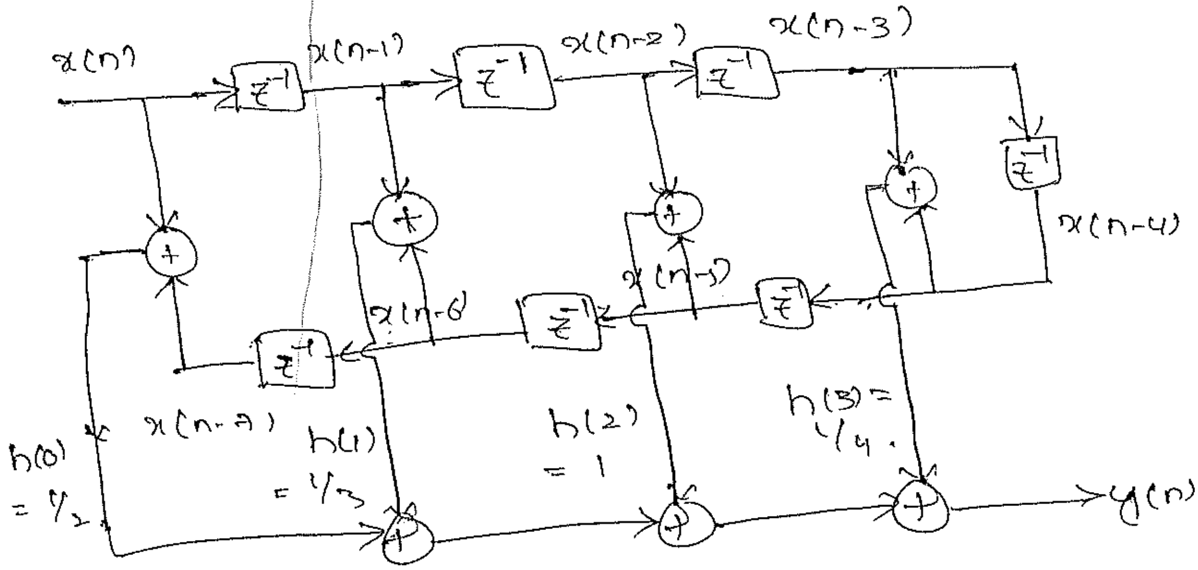
$$+ h(2)\left[z^{-2}X(z) + z^{-5}X(z)\right] + h(3)\left[z^{-3}X(z) + z^{-4}X(\right.$$

Applying I.Z.T and linearity prop

$$y(n) = h(0)\left[x(n) + x(n-7)\right] + h(1)\left[x(n-1) + x(n-6)\right] +$$

$$h(2)\left[x(n-2) + x(n-5)\right] + h(3)\left[x(n-3) + x(n-4)\right]$$

——② .



Total no.of multipliers used = $\frac{N}{2}$

$= 8/2 = 4$.

adders $= N-1 = 7$.

memory elements $= 7$.

Cascade:-

3. $H(z) = \left(1 + \frac{1}{2} z^{-1} + z^{-2}\right)\left(1/2 + z^{-1} + 1/2\, z^{-2}\right).$

$= H_1(z)\, H_2(z)$.

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = 1 + \frac{1}{2} z^{-1} + z^{-2}$$

$$Y_1(z) = X_1(z) + \frac{1}{2} z^{-1} X_1(z) + X_1(z)\, z^{-2} .$$

$$y_1(n) = x_1(n) + \frac{1}{2} x(n-1) + x(n-2) .$$

$$= \left[x_1(n) + x_1(n-2)\right] + \frac{1}{2} x_1(n-1) \quad ①$$

Hey.

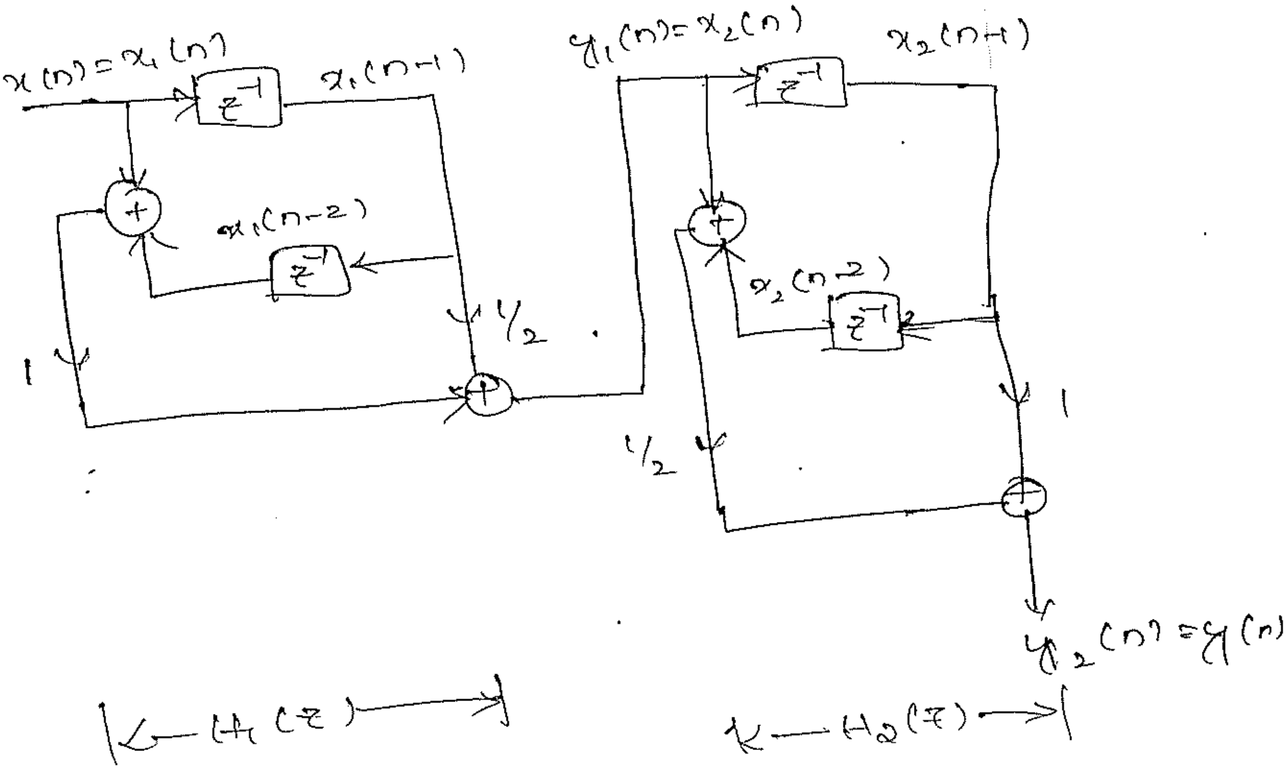
$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = \frac{1}{2} + z^{-1} + \frac{1}{2}z^{-2}.$$

$$Y_2(z) = \frac{1}{2}X_2(z) + z^{-1}X_2(z) + \frac{1}{2}z^{-2}X_2(z).$$

$$y_2(n) = \frac{1}{2}x_2(n) + x_2(n-1) + \frac{1}{2}x_2(n-2).$$

$$= \frac{1}{2}\left[x_2(n) + x_2(n-2)\right] + x_2(n-1)$$

$\longrightarrow ②$



4. FIR Filters

A filter is freq. selective sly. Digital filters are classified as finite duration unit impulse response (FIR) filters ~~are~~ or infinite duration unit impulse response (IIR) filters depending on the form of unit impulse response of the sly. So in the FIR sly the impulse response sequence is of finite duration. The IIR sly has an infinite no. of non-zero terms. IIR filters are usually implemented using recursive structures (feedback poles & zeros) & FIR filters are usually implemented using non-recursive structures (no F.B only zeros).

The response of the FIR filter depends only on the present and past i/p samples. Whereas for the IIR filter the present response is a function of the present and past values of the excitation as well as past values of the response.

Advantages of FIR filters over IIR filters:-

-> FIR filters are always stable.

-> FIR filters with exactly linear phase can easily be design.

-> FIR filters can be realize in both recursive and non-recursive structures.

-> FIR filters are free of limit cycle oscillations when implemented on a finite wordlength digital sly.

→ 3. excellent design methods are available for various kinds of FIR filter.

Disadvantages of FIR filter are as follows:-

→ The implementation of narrow transition band FIR filter is very costly as it requires more arithmetic operation & hardware components such as multiplier, adders, delay element.

→ Memory requirment & execution time are very high.

Comparision of IIR & FIR filter :-

IIR filter	FIR filter
→ All the infinite sample of impulse response are consider.	→ Only finite no. of samples of impulse response are considered.
→ The impulse response can't be directly converted to digital filter T.F.	→ The impulse response can be converted to digital filter T.F.
→ Linear phase characteristics can't be achieve.	→ Linear phase filters can be easily designed.
→ IIR filters are easily realized recursively.	→ FIR filters can be realized recursively & non-recursively.
→ The specifications include the desired characteristic for magnitude response only.	→ The specification include the derived characteristics for both magnitude & phase response.
→ The design involves design of analog filter & then transforming analog filter to digital filter.	→ The digital filter can be directly design to achieve the desired specifications.

- The round of noise in IIR filter is more.

→ Errors due to round o noise are less serious in FI filters mainly becau FB i not used.

Characteristics of FIR filters with linear phase

The T.F a FIR Causal filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

where $h(n)$ is the impulse response of the filter.

Now the freq response [F.T of $h(n)$] is given by

$$H(\omega) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} = H(e^{j\omega})$$

Since $H(\omega)$ is complex it can be expressed as

$$H(\omega) = \pm |H(\omega)| \angle H(\omega)$$

where $|H(\omega)|$ is the magnitude response and

$$H(\omega) = \pm |H(\omega)| e^{j\theta(\omega)}$$

Here $\theta(\omega)$ is phase response

We define the phase delay ~~T(P)~~ T_p & group delay T

of a filter as

$$T_p = \frac{-\theta(\omega)}{\omega}, \quad T_g = \frac{-d\theta(\omega)}{d\omega}$$

for FIR filters with linear phase we can define

$$\theta(\omega) = -\alpha\omega ; \quad -\pi \le \omega \le \pi$$

$$T_p = \frac{+\alpha\omega}{\omega} = +\alpha, \quad T_g = \frac{-d(-\alpha\omega)}{d\omega} = \alpha$$

Here $T_p = T_g = \alpha$ which means that α is independ

of frequency.

$$H(\omega) = \pm |H(\omega)| e^{j\theta(\omega)}$$

$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm|H(\omega)|e^{-j\alpha\omega}$$

$$\sum_{n=0}^{N-1} h(n)[\cos\omega n - j\sin\omega n] = \pm|H(\omega)|[\cos\alpha\omega - j\sin\alpha\omega] \longrightarrow ①$$

On equating the real part & imaginary part of the above eq'n we get

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm|H(\omega)|\cos\alpha\omega \longleftarrow ②$$

$$\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm|H(\omega)|\sin\alpha\omega. \longrightarrow ③$$

On dividing eq'n ③ ~~by~~ ②

$$\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\pm|H(\omega)|\sin\alpha\omega}{\pm|H(\omega)|\cos\alpha\omega}$$

$$\tan\omega n = \tan\alpha\omega$$

$$\sum_{n=0}^{N-1} h(n)\sin\omega n\cos\alpha\omega = \sum_{n=0}^{N-1} h(n)\sin\alpha\omega\cos\omega n.$$

$$\sum_{n=0}^{N-1} h(n)\cos\omega n\sin\alpha\omega - \sum_{n=0}^{N-1} h(n)\sin\omega n\cos\alpha\omega = 0.$$

$$\sum_{n=0}^{N-1} h(n)[\sin\alpha\omega\cos\omega n - \cos\alpha\omega\sin\omega n] = 0.$$

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha\omega - \omega n) = 0.$$

$$\boxed{\sum_{n=0}^{N-1} h(n)\sin(\alpha-n)\omega = 0} \longrightarrow ④$$

One solution of eq'n ④ exists when $\alpha = \frac{N-1}{2}$

and $h(n) = h(N-1-n)$ for $0 \le n \le N-1$

$h(n)\ \sin(\alpha - n)\omega.$

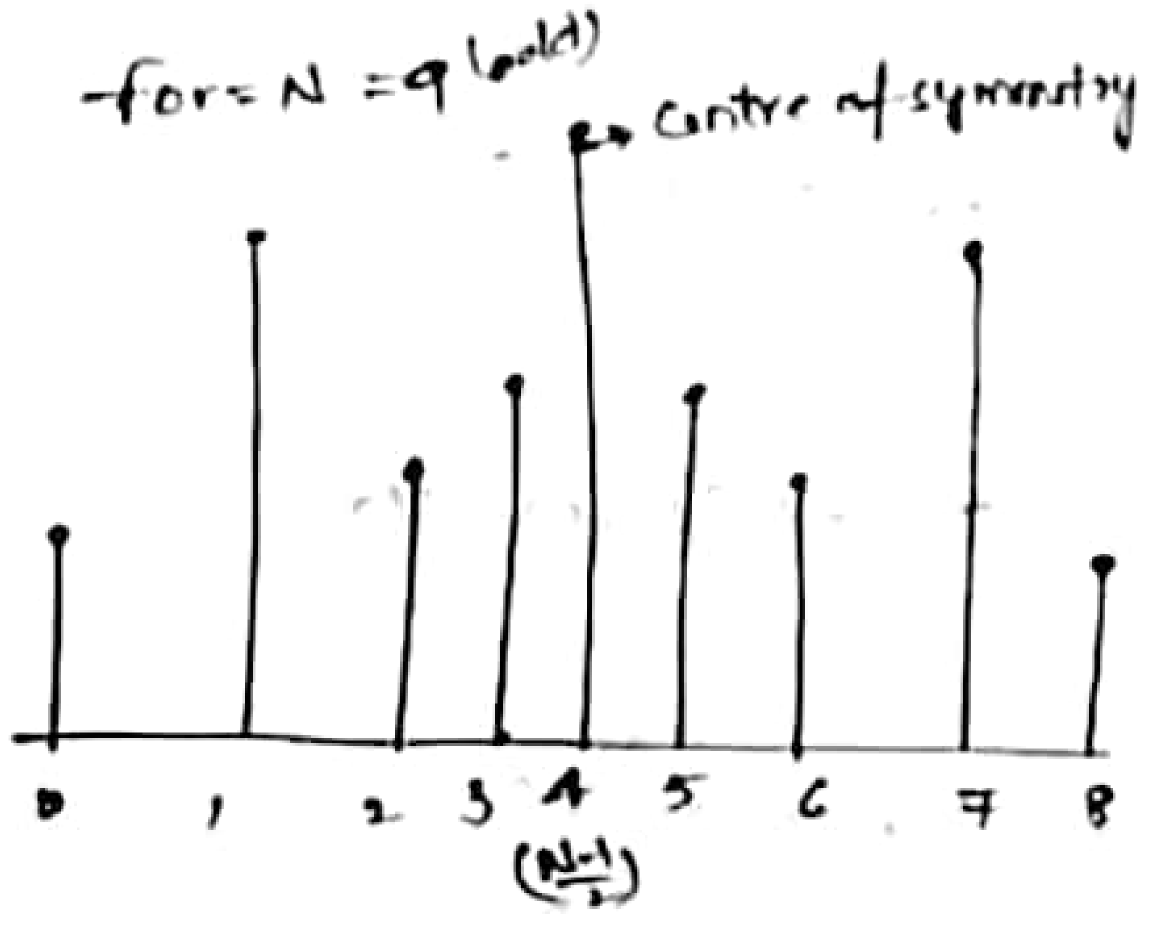
$$= h(n)\ \sin\left[\left(\frac{N-1}{2} - n\right)\omega\right)$$

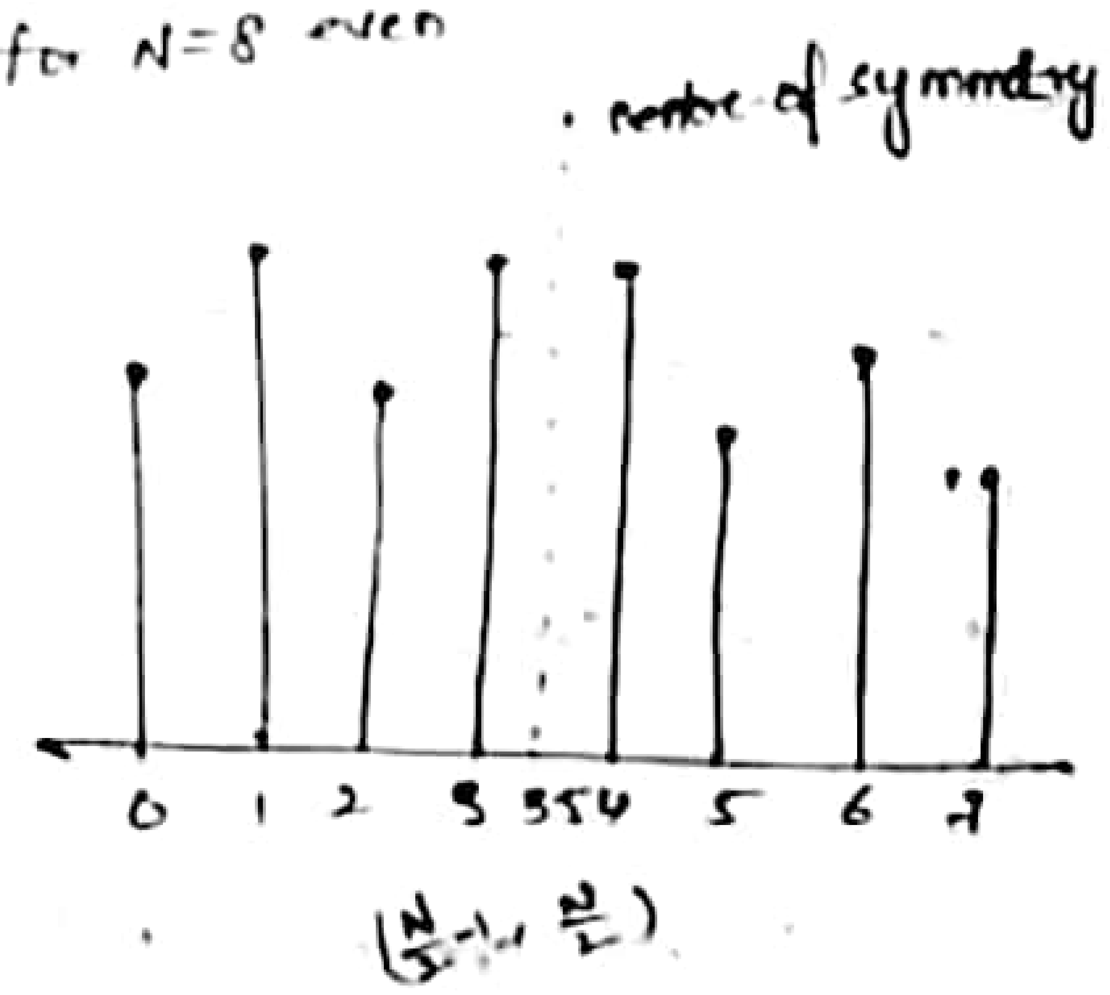
$$= h(n)\ \sin\left[\frac{N-1-n-n}{2}\ \omega\right]$$

$$= h(n)\ \sin\left[\frac{n-n}{2}\ \omega\right)$$

$= 0.$

This shows that FIR filters will have Constant phase and group delay when impulse response is symmetrical about $\alpha = \frac{N-1}{2}$. The impulse response satisfying the symmetric Condition $h(n) = h(N-1-n)$ for odd, even values of N is shown in fig1. when N=9 the center of the symmetry of sequence occurs at 4th sample & when N=8 the filter delay is 3 ½ samples.





<u>Case ii</u>:- If only Constant group delay is required & not the phase delay

$$\theta(\omega) = \beta - \alpha\omega$$

$$\tau_p = -\frac{\theta(\omega)}{\omega} = \frac{-(\beta - \alpha\omega)}{\omega} = \alpha - \beta/\omega;$$ it is not a Constant

$$\tau_g = \frac{-d\ \theta(\omega)}{d\omega} = \frac{-d(\beta - \alpha\omega)}{d\omega} = \alpha;$$ It is Constant

$$H(\omega) = \pm |H(\omega)| e^{j\theta(\omega)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(\omega)| e^{j(\beta-\alpha\omega)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\left[\cos\omega n - j\sin\omega n\right] = \pm |H(\omega)| \left[\cos(\beta-\alpha\omega) + j\sin(\beta-\alpha\omega)\right]$$

equating real and imaginary parts

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm H(\omega)\cos(\beta-\alpha\omega) \longrightarrow ⑥$$

$$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm H(\omega)\sin(\beta-\alpha\omega) \longrightarrow ⑦$$

Eq ⑦/⑥ ⇒
$$\frac{-\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\pm H(\omega)\sin(\beta-\alpha\omega)}{\pm H(\omega)\cos(\beta-\alpha\omega)}$$

$$-\sum_{n=0}^{N-1} h(n)\sin\omega n\left[\cos(\beta-\alpha\omega)\right] = \sum_{n=0}^{N-1} h(n)\cos\omega n\left[\sin(\beta-\alpha\omega)\right]$$

$$\sum_{n=0}^{N-1} h(n)\cos\omega n\sin(\beta-\alpha\omega) + \sum_{n=0}^{N-1} h(n)\sin(\omega n)\cos(\beta-\alpha\omega) = 0$$

$$\sum_{n=0}^{N-1} h(n)\left[\cos(\omega n)\sin(\beta-\alpha\omega) + \sin(\omega n)\cos(\beta-\alpha\omega)\right] = 0 \qquad \therefore \sin(A+B)$$

$$\sum_{n=0}^{N-1} h(n)\sin(\omega n+\beta-\alpha\omega) = 0$$

$$\boxed{\sum_{n=0}^{N-1} h(n)\sin(\omega(n-\alpha)+\beta) = 0} \text{ (or) } \sum_{n=0}^{N-1} h(n)\sin(\beta-(\alpha-n)\omega) = 0$$

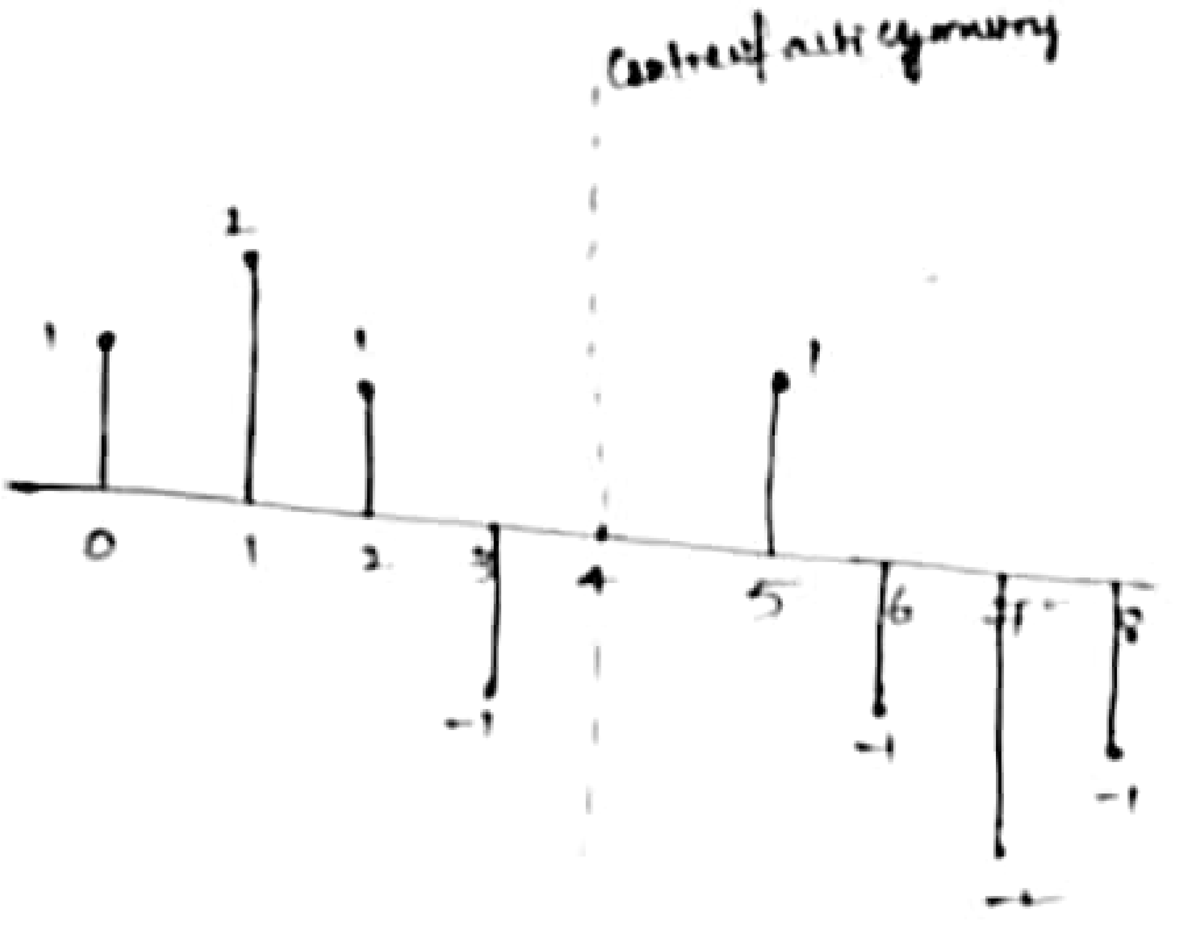
If $\beta = \pi/2$

$$\sum_{n=0}^{N-1} h(n)\sin(\pi/2-(\alpha-n)\omega) = 0 \Rightarrow \boxed{\sum_{n=0}^{N-1} h(n)\cos(\alpha-n)\omega = 0}$$

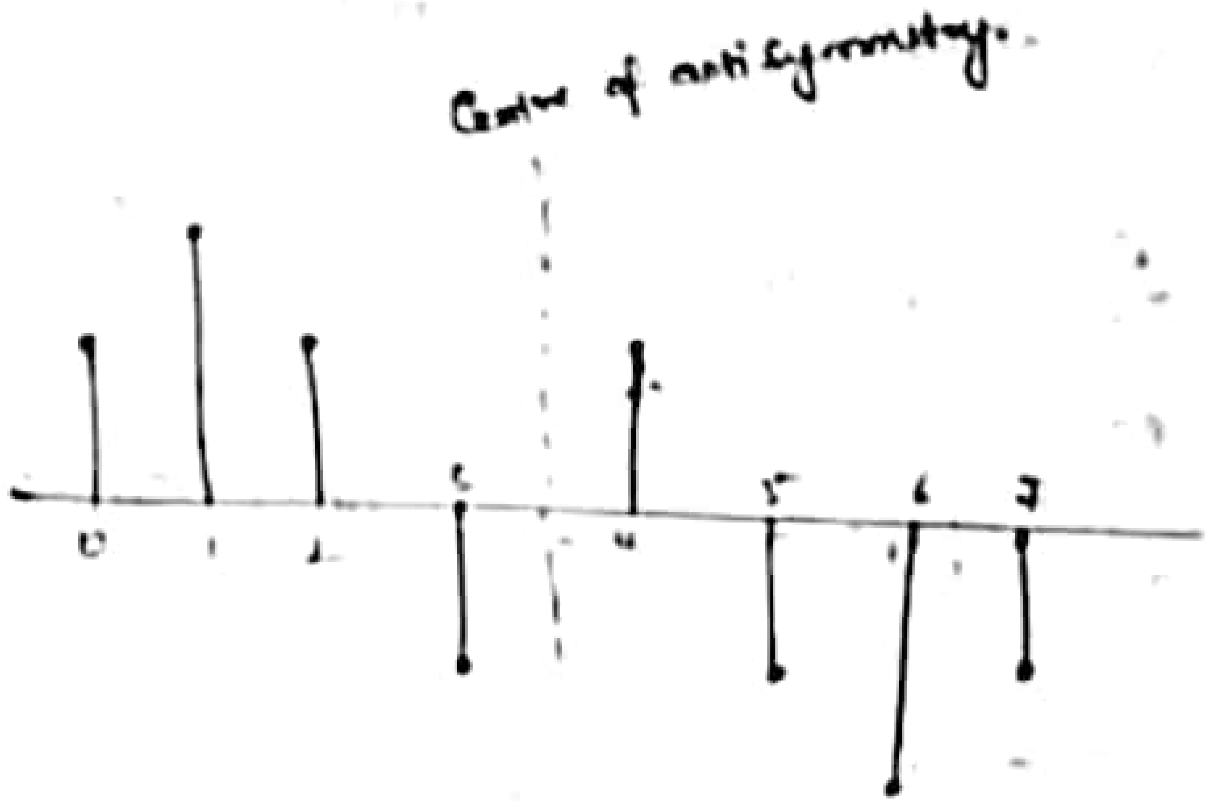
This eq'n will be satisfy when $\alpha = \frac{N-1}{2}$ and $h(n) = -h(N-1-n)$
This shows that FIR filters have Constant group delay $\tau_g = \alpha$ and τ_p = not Constant when the impulse response is anti symmetrical about $\alpha = \frac{N-1}{2}$.

Ex: The impulse response satisfying the anti symmetric condition is shown in the following figures when N=9 the centre of anti symmetry occurs at 4th sample and when N=8 the centre of anti symmetry occurs at 3½ sample.

For N=9



for N=8



Frequency response of linear phase FIR filter:-

→ The frequency response of the filter is the F.T of its impulse response.

→ If h(n) is the impulse response of the s/y then the freq response of the s/y is denoted by $H(e^{j\omega})$ (or) $H(\omega)$

→ $H(\omega)$ is a complex fu'n of freq ω and the can has it can be expressed as magnitude fu'n $|H(j\omega)|$ & phase fu'n $\angle H(j\omega)$

→ Depending on the value of N{odd (or) even} and the type of symmetry response of the filter impulse response sequence (symmetry (-Anti symmetry), there are following 4 types of impulse response for Lin phase FIR filter

→ freq. response of linear phase FIR filter when imp response is symmetrical and N is odd:-

Let h(n) be the impulse response of the sly. The freq. response of the sly H(ω) is given by

$$H(\omega) = H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{\infty} h(n) e^{-j\omega n}.$$

When N odd h(n) is symmetric Centre of symmetry is at $\frac{N-1}{2}$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

$m = N-1-n \; ; \; n = N-1-m$

$n = \frac{N+1}{2} \; ; \; m = N-1 - \frac{N+1}{2} \quad \frac{N-3}{2}$

$n = N-1 \; ; \; m = N-1-(N-1) = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-m)\, e^{-j\omega(N-1-m)}$$

put m = n

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} e^{j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{n}} h(n) e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2} - n\right)} + e^{-j\omega\left(N-1-n-\left(\frac{N-1}{2}\right)\right)} \right] \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2} - n\right)} + e^{-j\omega\left(\frac{N-1}{2} - n\right)} \right] \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right)+\sum_{n=0}^{\frac{N-3}{2}} h(n)\left[2\cos\omega\left[\frac{N-1}{2}-n\right]\right]\right]$$

Let $k = \frac{N-1}{2} - n$; $n = \frac{N-1}{2} - k$.

$n = 0 ; k = \frac{N-1}{2}$; $n = \frac{N-3}{2}$, $k = \frac{N-1}{2} - \frac{N-3}{2} = \frac{N-1-N+3}{2} = 1$.

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right)+\sum_{k=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2}-k\right]\cos\omega k\right]$$

put $k = n$.

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right)+\sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2}-n\right]\cos\omega n\right]$$

The magnitude fu'n of $H(\omega)$ is given by

$$|H(\omega)| = h\left(\frac{N-1}{2}\right)+\sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos\omega n$$

The phase fu'n of $H(\omega)$ is given by

$$\angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega\alpha.$$

freq. response of Linear phase FIR filter when impu response is Symmetrical and N ~~odd~~ even :-

Proof :- freq. response of Linear phase FIR filter. With impulse response h(n) & N samples.

$$H(\omega) = H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}.$$

For symmetric impulse response with even no. of Samp the centre of symmetry lies b/n. $n = \frac{N-1}{2}$ & $n = \frac{N}{2}$

Hence $H(\omega)$ is expressed as

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)e^{-j\omega n}.$$

Let $m = N-1-n$; $n = N-1-m$

$H(\omega)$: $n = \frac{N}{2}$, $m = N-1-\frac{N}{2} = \frac{N-2}{2} = \frac{N}{2}-1$

$n = N-1$, $m = N-1-N+1 = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m) e^{-j\omega(N-1-m)}$$

put m=1

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n) e^{-j\omega(N-1-n)}$$

The impulse response is symmetric $h(N-1-n) = h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left(\sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} e^{j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)} \right)$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{+j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left[N-1-n-\left(\frac{N-1}{2}\right)\right]} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left[\frac{N-1}{2}-n\right]} \right]$$

$$~~H(\omega)~~ = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2\cos\omega\left[\frac{N-1}{2}-n\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2\cos\omega\left[\frac{N}{2}-n-\frac{1}{2}\right]$$

Let $k = \frac{N}{2}-n$; $n = \frac{N}{2}-k$

$n = 0$; $k = \frac{N}{2}$.

$n = \frac{N}{2}-1$; $k = \frac{N}{2}-\frac{N}{2}+1 = 1$

Put $k = n$.

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ \sum_{n=1}^{\frac{N}{2}} h\left(\frac{N}{2}-n\right) 2\cos\omega\left[n-\frac{1}{2}\right] \right\}$$

$$|H(\omega)| = \sum_{n=0}^{\frac{N}{2}} 2h\left(\frac{N}{2}-n\right) \cos\omega\left[n-\frac{1}{2}\right]$$

$\angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega\alpha$. where $\alpha = \frac{N-1}{2}$.

Frequency response of Linear phase FIR-filter when imp response is Antisymmetric and N is odd :-

Proof: freq. response of linear phase FIR-filter with impu response h(n) of N-samples

$$H(\omega) = H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}.$$

anti symm.

- for a impulse response with N odd samples, the centre of symmetry is at $\frac{N-1}{2}$.

$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}.$$

If impulse response is anti symmetric $h\left(\frac{N-1}{2}\right) = 0$.

$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} h(n) e^{-j\omega n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

Let $m = N-1-n$, $n = N-1-m$.

$n=0 \Rightarrow m = N-1$

$n = \frac{N+1}{2} \Rightarrow m = N-1-\frac{N+1}{2} = \frac{N-3}{2}$

$n = N-1 \Rightarrow m = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m) e^{-j\omega(N-1-m)}$$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) e^{-j\omega(N-1-m)}$$

The impulse response is anti symmetry. $h(N-1-n) = -h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} - \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-m)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{+j\omega\left[\frac{N-1}{2}-n\right]} - e^{-j\omega\left[N-1-n-\left(\frac{N-1}{2}\right)\right]}\right]\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(\frac{N-1}{2}-n\right)}\right]$$

Let $k = \frac{N-1}{2} - n$; $n = \frac{N-1}{2} - k$.

$n=0 \Rightarrow k = \frac{N-1}{2}$

$n = \frac{N-3}{2}$, $k = \frac{N-1}{2} - \frac{N-3}{2} = 1$

$$= e^{-j\omega\left[\frac{N-1}{2}\right]} \sum_{k=1}^{\frac{N-1}{2}} \cancel{h(n)} \left[h\left[\frac{N-1}{2}-n\right] 2j\sin\omega\left[\overset{k}{\frac{N-1}{2}-n}\right]\right]$$

→ . system properties.

→ ~~D~~ Impulse response, step response.

→ DIT & FFT

→ Circular Convolution

→ Difference b/w analog & digital filter,

→ direct form 1.

$$|H(j\omega)| = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right] j \sin \omega n \; .$$

$$j = e^{j\pi/2}$$

$$H(\omega) = e^{j\pi/2} e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin \omega n.$$

$$= e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \sin \omega n.$$

$$|H(\omega)| = \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \sin \omega n.$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega\left[\frac{N-1}{2}\right] = \beta - \omega\alpha$$

where $\beta = \frac{\pi}{2}$, $\alpha = \frac{N-1}{2}$

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Frequency response of Linear phase FIR-filter when impul
response is Antisymmetric and N is even:-

Proof:- freq. Response of Linear phase FIR-filter with impulse response h(n) in N sample.

$$H(\omega) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

Impulse response is Anti Symmetric and N-even the centre of antisymmetry lies b/n $\frac{N}{2}-1$ & $\frac{N}{2}$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)\, e^{-j\omega n}$$

Let $m = N-1-n$; $n = N-1-m$

$n = \frac{N}{2}$; $m = N-1-\frac{N}{2} = \frac{N}{2}-1$

$n = N-1$; $m = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m)\, e^{-j\omega(N-1-m)}$$

put $m = n$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

The impulse response is Anti Symmetric $h(N-1-n) = -h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} -h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(N-1-n-\left(\frac{N-1}{2}\right)\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2j \sin\omega\left[\frac{N-1}{2}-n\right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2j \sin\omega\left[\frac{N}{2} - n - \frac{1}{2}\right].$$

Let $k = \frac{N}{2} - n \Rightarrow n = \frac{N}{2} - k$.

$n = 0 \; ; \; k = \frac{N}{2}$

$n = \frac{N}{2} - 1 \; ; \; k = 1$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{k=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - k\right) j\sin\omega\left[k - \frac{1}{2}\right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) e^{j\pi/2} \sin\omega\left[n - \frac{1}{2}\right]$$

$$H(\omega) = e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\omega\left(n - \frac{1}{2}\right)$$

$$\therefore |H(\omega)| = \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\omega\left(n - \frac{1}{2}\right).$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) = \beta - \omega\alpha.$$

Where $\beta = \frac{\pi}{2}$, $\alpha = \frac{N-1}{2}$.

Design Techniques for Linear Phase FIR filters:

The well known methods of designing FIR filter as follows.

1. Fourier series method.
2. Window method.
3. Frequency sampling method.

Fourier series method:-

The freq. response $H(e^{j\omega})$ or $H(\omega)$ of a system is periodic in 2π from F.S analysis we know that any periodic fun can be expressed as a linear combination of complex exponential.

∴ The desired freq. response of an FIR filter can be represented by the F.S

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n} \text{ ——— } ①$$

where the fourier coefficients $h_d(n)$ are the desired impulse response sequence of the system. The samples of $h_d(n)$ can be determine by using the eq'n (which is inverse FT $H_d(e^{j\omega})$)

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega \text{ ——— } ②$$

The impulse response $h_d(n)$ from eq'n ② is an infinite duration sequence

For FIR filters we truncate this infinite impulse response to a finite duration sequence of length 'N' where 'N' is odd

$$h(n) = \begin{cases} h_d(n) & ; \text{ for } n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right) \\ 0 & , \text{ otherwise} \end{cases}$$

Taking Z.T of h(n) we get

$$Z.T[h(n)] = H(z) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n) z^{-n}$$

$$= \sum_{n=-\frac{N-1}{2}}^{-1} h(n) z^{-n} + h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{-n} \text{ ——— } ③$$

$$= \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{n} + h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{-n} \text{ ——— } ④$$

Impulse response in Symmetric

$$h(-n) = h(n)$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \text{ ——— } ⑤$$

This transfer fu'n of the filter H(z) represents a non-Causal filter (due to the presence of +ve power subject).

So, the transfer fu'n H(z) is not physically realizable

Realizability Can be brough by multiplying eq'n ⑤ by

where $\frac{N-1}{2}$ is delay in samples.

Hence we see that Causality is brought about by multiplying the transfer fu'n by the delay factor $\alpha = \frac{N-1}{2}$

this modification doesn't effect the amplitude response of the amplitude response of the filter.

However the abrupt truncation of the FS result in oscillations in the pass & stop band

These oscillation are due to the slow convergence of the particurly near the point of dis continuity. These effect is known as gibbs phenomenon the undesirable oscillation Can be reduced by multiplying the desired freq. response Coefficient by an appropriate window fu'n.

Design procedure for digital FIR filter by Fourier series method:

→ The specification of Digital FIR filter are

* The desired freq. respon $H_d(jw)$
* The Cutoff freq. w_c for Lowpass & HPF and w_{c1} & w_{c2} for bandpass & band stop filters

Note:- If analog filter cut-off freq. F_c and Sampli freq. F_s are specified then Calculate the cut-freq. of digital filter w_c using the eqn $\boxed{w_c = \frac{2\pi f_c}{F_s}}$

* The no. of samples of impulse response 'N'.

→ Determine the desired impulse response $h_d(n)$ by $h_d(n) = \frac{1}{2\pi}$ taking inverse F.T of the desired freq. response $H_d(e^{jw})$.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{jw})\,dw$$

→ Calculate N samples of $h_d(n)$ for $n = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$

And from the impulse response $h(n) = h_d(n) = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$ with n = 0 and so

Note:- The impulse response is symmetric $h(-n) = h(n)$

Hence it is sufficient we calculate $h(n)$ for $n = 0$ to $\frac{N-1}{2}$

→ Take z.T of impulse response to get $\frac{N-1}{2}$ the non-causal transfer function of FIR filter $H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)} h(n) z^{-n}$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) \{z^{-n} + z^{n}\}$$

→ Convert the non-causal T.F $H(z)$ to causal T.F $H'(z)$ by multiplying $H(z)$ by $z^{-\left(\frac{N-1}{2}\right)}$

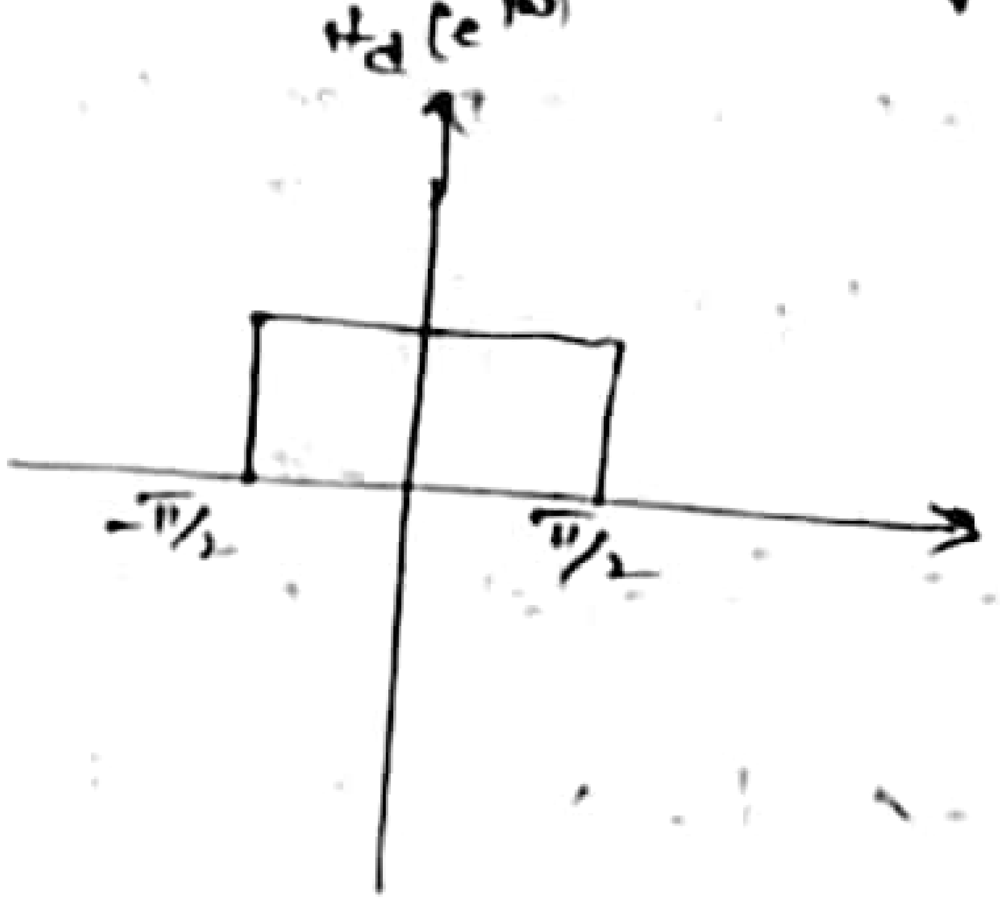
$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) [z^{n} + z^{-n}] \right]$$

→ Draw a suitable structure for realization of FIR filter.

* Design an ideal LPF with freq. response

$$H_d(e^{j\omega}) = \begin{cases} 1 \text{ for } & -\pi/2 \le \omega < \pi/2 \\ 0 \text{ for } & \pi/2 \le |\omega| \le \pi . \end{cases}$$, find the values

of h(n) for N = 11 , find H(z) & plot the magnitude response.



Sol:-

$$H_d(e^{j\omega}) = \begin{cases} 1 \text{ for } & -\pi/2 \le \omega < \pi/2 \\ 0 \text{ for } & \pi/2 \le |\omega| \le \pi \end{cases}$$

N = 11.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega$$

$$\boxed{\alpha = \frac{N-1}{2}}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{jn}\right]$$

$$\boxed{h_d(n) = \frac{1}{\pi n} \sin\left(\frac{\pi n}{2}\right)}$$ $n = -\alpha$ to α.

$$h(n) = h_d(n) \Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right)} = -5 \text{ to } 5$$

$$h(n) = h(-n)$$

$h(0) = \frac{0}{0}$ = underterminat

By using L- hospital rule $\left(\lim_{\theta \to 0} \frac{\sin n\theta}{\theta} = n \quad \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \right)$

$$h(0) = h_d(0) = \underset{n \to 0}{Lt} \frac{\sin\frac{\pi n}{2}}{\pi n} = \frac{1}{2}$$

$$h(1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.318 = h(-1)$$

$$h(2) = \frac{\sin \pi}{2\pi} = 0 = h(-2)$$

$$h(3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = \cancel{-0.079}\ -0.106 = h(-3)$$

$$h(4) = \frac{\sin\frac{4\pi}{2}}{4\pi} = 0 = h(-4)$$

$$h(5) = \frac{\sin\left(\frac{5\pi}{2}\right)}{5\pi} = 0.065 = h(-5)$$

$$\boxed{H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]}$$

$$= \frac{1}{2} + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right]$$
$$+ h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right]$$

$$= \frac{1}{2} + 0.318(z^{1} + z^{-1}) + 0 - 0.106\left[z^{3} + z^{-3}\right] + 0.0636\left[z^{5} + z^{-5}\right]$$

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= z^{-5} H(z)$$

$$= z^{-5}\left[0.5 + 0.318(z^{1} + z^{-1}) - 0.106\left(z^{3} + z^{-3}\right] + 0.0636\left[z^{5} + z^{-5}\right]\right]$$

$$= 0.5z^{-5} + 0.318z^{-4} + 0.318z^{-6} - 0.106z^{-2} - 0.106z^{-8}$$
$$+ 0.0636 + 0.0636z^{-10}$$

$$= 0.0636 - 0.106z^{-2} + 0.318z^{-4} + 0.5z^{-5} + 0.318z^{-6}$$
$$- 0.106z^{-8} + 0.0638z^{-10}$$

h(n) = filter Coefficients.

$h(n) = h(N-1-n)$

$h(0) = h(10) = 0.0636$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

$h(4) = h(6) = 0.318$

$h(5) = h(5) = 0.5$

$\boxed{\text{odd}, N=11}$

magnitude response of $H(e^{j\omega})$.

$$|H(e^{j\omega})| = |H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos\omega n.$$

$$|H(e^{j\omega})| = h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega$$

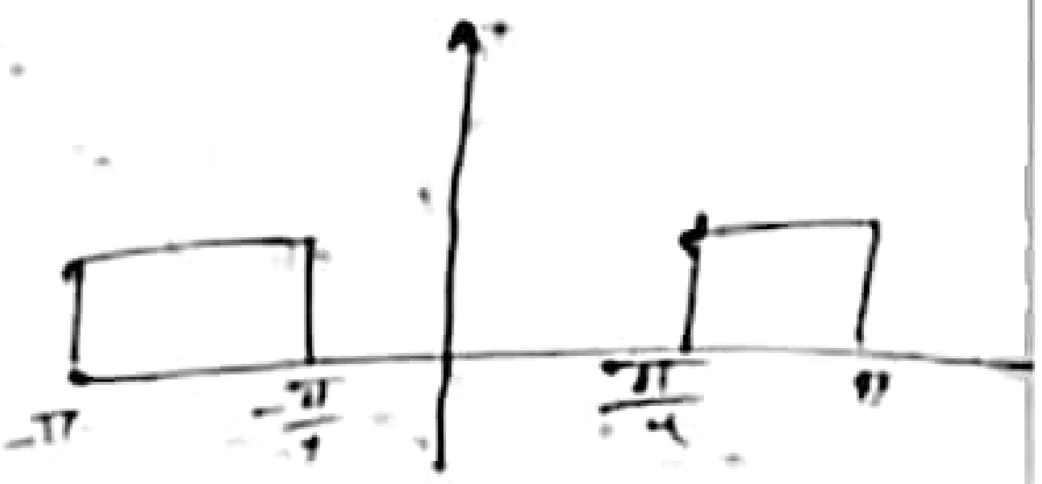
$$+ 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.5 + 2(0.318)\cos\omega + 2(-0.106)\cos 3\omega + 2(0.0636)\cos 5\omega$$

$$= 0.5 + 0.636\cos\omega - 0.212\cos 3\omega + 0.1272\cos 5\omega$$

ω	0	10	20	30	40	50	60	70	80	90	100	110	120
$\lvert H(e^{j\omega})\rvert$	0	0.056	1.07	-	-0.21	-0.212	0.77	0.21	1.79	-6	-14.56	-31.8	-20.6

* Design an ideal HPF with freq. response $H_d(e^{j\omega}) = 1$ for $\frac{\pi}{4} \le |\omega| \le \pi$; 0 for $|\omega| \le \pi/4$. Find the values of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.



Sol:-

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi}$$

$$* \left| ~~= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{\pi}~~ = \frac{1}{2\pi}\left[\frac{e^{j\pi n} - e^{j\frac{\pi}{4}n}}{jn}\right]^{*}\right.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{-\frac{\pi}{4}}\right] + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\frac{\pi n}{4}} - e^{-j\pi n}\right] + \frac{1}{2\pi jn}\left(e^{j\pi n} - e^{j\frac{\pi n}{4}}\right)$$

$$= \frac{1}{2\pi jn}\left[e^{j\pi n} + e^{-j\frac{\pi n}{4}} - e^{j\frac{\pi n}{4}} - e^{-j\pi n}\right]$$

$$= \frac{1}{2\pi jn}\left[2j\sin\pi n + 2j\sin\left(\frac{\pi n}{4}\right)\right]$$

$$= \frac{1}{\pi n}\left[\sin\pi n + \sin\left(\frac{\pi n}{4}\right)\right] \qquad n = -\infty \text{ to } \infty$$

$$h(n) = h_d(n)\Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right)} = -5 \text{ to } 5$$

$$h(n) = h(-n)$$

$$\times \left| h(0) = \frac{1}{\pi(0)}\right. [\sin \quad = \lim_{n\to 0}\left(\frac{\sin\pi n + \sin\left(\frac{\pi n}{4}\right)}{n\pi}\right)$$

$$h(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\frac{\pi}{4}} 1\, d\omega + \frac{1}{2\pi}\int_{+\pi/4}^{\pi} d\omega$$

$$= \frac{1}{2\pi}\left[\omega\Big|_{-\pi}^{-\pi/4}\right] + \frac{1}{2\pi}\left[\omega\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi}{4}+\pi\right] + \frac{1}{2\pi}\left[\pi - \frac{\pi}{4}\right] = \frac{3}{4}$$

$$h(1) = h(-1) = \frac{1}{\pi}\left[\sin\pi - \sin\frac{\pi}{4}\right] = -0.225.$$

$$h(2) = h(-2) = -0.159$$

$$h(3) = h(-3) = \cancel{-0.075} - 0.075$$

$$h(4) = h(-4) = -0.056 = 0$$

$$h(5) = h(-5) = \cancel{-0.045} = 0.045$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= \frac{3}{4} + h(1)\left[z + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right]$$

$$+ h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right]$$

$$= \frac{3}{4} - 0.225\left[z + z^{-1}\right] - 0.159\left[z^{2} + z^{-2}\right] - 0.075\left[z^{3} + z^{-3}\right].$$

$$0.045\left[z^{5} + z^{-5}\right]$$

Realizable filter T.F :-

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z).$$

$$= z^{-5}\left[H(z)\right]$$

$$= 0.75z^{-5} - 0.225z^{-4} - 0.225z^{-6} - 0.15z^{-3} - 0.159z^{-7} - 0.075z^{-2}$$

$$-0.075z^{-8} + 0.045 + 0.045z^{-10}$$

$$= 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.225z^{-4} + 0.75z^{-5}$$

$$-0.225z^{-6} - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10}$$

Causal filter Coefficients are

$h(0) = h(10) = 0.045$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.075$

$h(3) = h(7) = -0.159$

$h(4) = h(6) = -0.225$

$h(5) = 0.75$

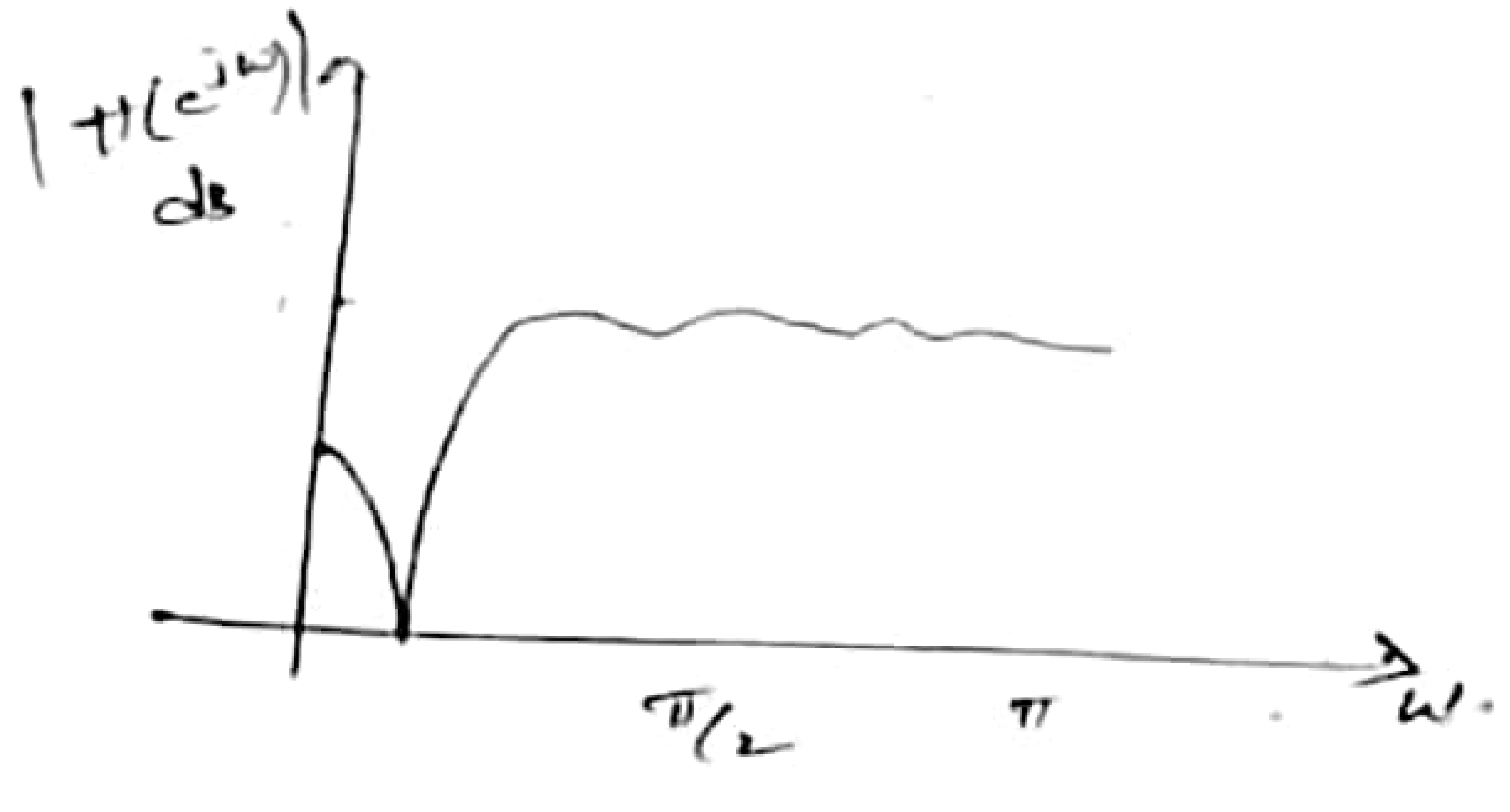
$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right] \cos\omega n$$

$$= h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega$$
$$+ 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 5$$

$\underset{\omega\leftarrow\pi}{\omega} = 10^{\circ}$

ω	0	10	20	30	40	50	60	70	80	90	10
$\lvert H(e^{j\omega})\rvert$	-22dB	-23.62	-41.3	-18.2	-9.16	-4.2	-1.1	-0.504	0.95	0.157	-0



* Design an ideal band pass filter with freq. response $H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0 & \text{otherwise} \end{cases}$ find the values of $h(n)$ for $N=11$ & plot freq. response.

sol: $h_d(n) = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{-\pi/4} 1\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\frac{3\pi}{4}} e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn} \Big|_{-3\pi/4}^{-\pi/4} \right] + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn} \Big|_{\pi/4}^{3\pi/4} \right]$$

$$h_d(n) = \frac{1}{2\pi}\left(\frac{e^{-j\frac{n\pi}{4}} - e^{-j\frac{3n\pi}{4}}}{jn} \right) + \frac{1}{2\pi jn}\left[e^{j\frac{3n\pi}{4}} - e^{j\frac{n\pi}{4}} \right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\frac{n\pi}{4}} - e^{-j\frac{3n\pi}{4}} + e^{j\frac{3n\pi}{4}} - e^{j\frac{n\pi}{4}} \right]$$

$$= \frac{1}{2\pi jn}\left[2j\sin\left(\frac{3n\pi}{4}\right) - 2j\sin\left(\frac{n\pi}{4}\right) \right]$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{3n\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right) \right]$$

$$h(n) = h_d(n) \Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right)} = -5 \text{ to } 5$$

$$h(0) = \underset{n\to 0}{\text{Lt}} \frac{1}{\pi n} \frac{\sin\frac{3n\pi}{4}}{\frac{3n\pi}{4} \cdot \frac{4}{3}} - \underset{n\to 0}{\text{Lt}} \frac{\sin\frac{n\pi}{4}}{\frac{n\pi}{4} \times 4}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$h(1) = h(-1) = \frac{1}{\pi}\left[\sin\frac{3\pi}{4} - \sin\frac{\pi}{4} \right] = 0$

$h(2) = h(-2) = -0.3183$

$h(3) = h(-3) = +0$

$h(4) = h(-4) = 0$

$h(5) = h(-5) = 0$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n} \right]$$

$$= 0.5 + h(2)\left(z^{n} + z^{-n} \right) = 0.5 - 0.3183\left(z^{-n} + z^{n} \right)$$

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= z^{-5}\left(0.5 + 0.3183\left(z^{-2} + z^{2} \right) \right)$$

$$= 0.5z^{-5} - 0.3183z^{-7} - 0.3183z^{-3}$$

$= -0.3183z^{-3} + \cancel{0.3183z^{-7}}\ 0.5z^{-5} - 0.3183z^{-7}$

$h(0) = h(10) = 0.5$

$h(1) = h(9) = 0$

$h(2) = h(8) = \cancel{-0.3183}\ 0$

$h(3) = h(7) = -0.3183$

$h(4) = h(6) = 0$

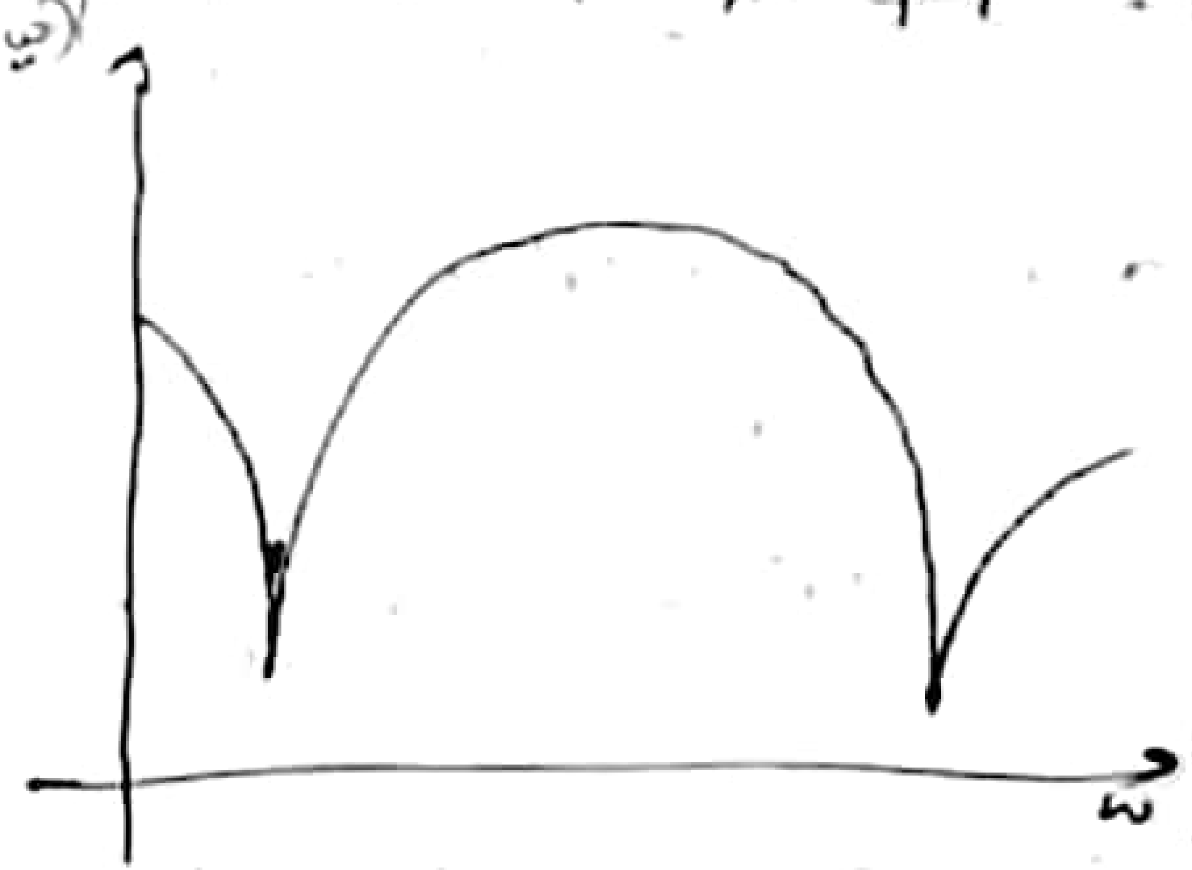
$h(5) = 0.5$

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos \omega n$$

$$= 0.5 + 2h(3)\cos 2\omega.$$

$$|H(e^{j\omega})| = 0.5 - 0.6366 \cos 2\omega.$$

ω	0	20	30	45	60	75	90	105	120	135	150	160	180
$20\log\|H(e^{j\omega})\|$	-17.3	-35.17	-14.8	-16.02	-1.74	0.4346	1.11	0.4346	-1.74	6.02	-14.6	~~32~~ 17	-17.3

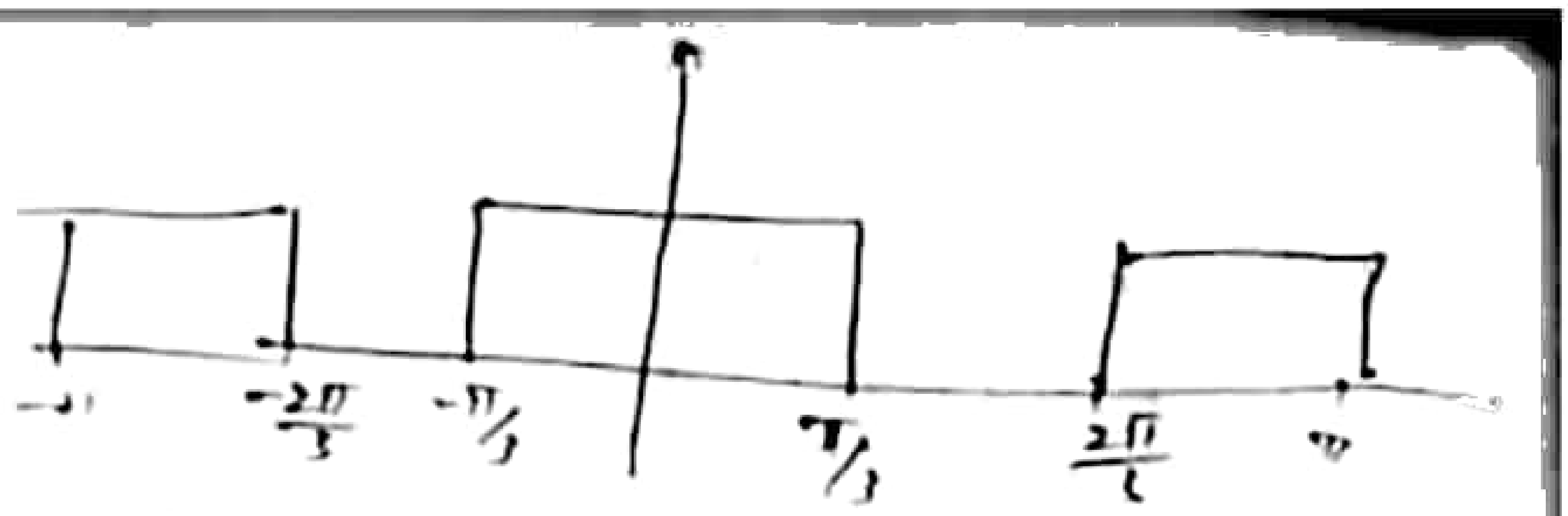


* Design an ideal band Reject filter with a desired freq. response $H(e^{j\omega}) = 1$ for $|\omega| \le \pi/3$ & $|\omega| \ge 2\pi/3$
$= 0$ for otherwise.

find $h(n)$ for $N=11$. find $H(z)$ plot the magnitude response.

Sol:- $h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$

$\|\omega\| \le \pi/3$	$\|\omega\| \ge 2\pi/3$
$\omega \le \pi/3$	$\omega \ge 2\pi/3$
$-\omega \le \pi/3$	$-\omega \ge 2\pi/3$
$\omega \le -\pi/3$	$\omega \ge -2\pi/3$



$$= \frac{1}{2\pi}\int_{-\pi}^{-\frac{2\pi}{3}} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\frac{2\pi}{3}}^{\pi} e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{-\frac{2\pi}{3}} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi/3}^{\pi/3} + \frac{e^{j\omega n}}{jn}\Big|_{\frac{2\pi}{3}}^{\pi} \right]$$

$$= \frac{1}{2\pi jn}\left[\left(e^{j\pi n} - e^{-j\pi n}\right) + \left(e^{\frac{jn\pi}{3}} - e^{-\frac{j\pi n}{3}}\right) + \left(e^{jn\frac{\pi}{3}} - e^{-\frac{2n\pi}{3}}\right) \right]$$

$$= \frac{1}{\pi n}\left[\sin n\pi + \sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3} \right)$$

Step2 $h(n) = h(-n)$

$N = 11$

$h(n) = h_d(n)\big|_n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N+1}{2}\right) = -5 \text{ to } 5.$

$h(0) = \lim_{n\to 0}\left[\frac{1}{\pi n}\left(\sin n\pi + \sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3}\right)\right]$

$= 1 + \frac{1}{3} - \frac{2}{3} = \frac{2}{3}.$

$h(1) = h(-1) = 0$

$h(2) = h(-2) = 0.275$

$h(3) = h(-3) = 0$

$h(4) = h(-4) = -0.317$

$h(5) = 0.$

Step3:- non-causal.

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)[z^{n} + z^{-n}]$$

$$H(z) = h(0) + h(1)[z^{-1} + z^{1}] + h(2)[z^{2} + z^{-1}]$$

$$+ h(3)[z^{3} + z^{-1}] + h(4)[z^{4} + z^{-4}] + h(5)[z^{5} + z$$

$$H(z) = 0.66 + 0.275(z^{-2} + z^{2}) + 0 - 0.317(z^{-4} + z^{4}) + 0$$

$$H(z) = 0.66 + 0.275(z^{-2} + z^{2}) - 0.317(z^{-4} + z^{4})$$

Step4 Causal.

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$H'(z) = z^{-5}\left[0.66 + 0.275(z^{-2} + z^{2}) - 0.317(z^{-4} + z^{4}\right.$$

$$= -0.317z^{-1} + 0.275z^{-3} + 0.66z^{-5} + 0.275z^{-7} - 0.317z$$

Step5:- Causal filter coefficient

$h(n) = h(N-1-n)$

$h(0) = h(10) = 0$

$h(1) = h(9) = -0.317$

$h(2) = h(8) = 0$

$h(3) = h(7) = 0.275$

$h(4) = h(6) = 0$

$h(5) = 0.66.$

step 6 :- Magnitude Response

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \cos \omega n.$$

$$= h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega + 2h(1)\cos 4\omega$$
$$+ 2h(0)\cos 5\omega.$$

$$|H(e^{j\omega})| = 0.66 + 0.55\cos 2\omega - 0.274\cos 4\omega$$

b. Design of FIR filters using window.

The procedure for designing FIR filter using window is

- → Choose the desired freq. Response of the filter $H_d(e^{j\omega})$
- → Take I.F.T of $H_d(e^{j\omega})$ to obtain the desired impulse Response $h_d(n)$
- → Choose a window sequence $w(n)$ & multiply $h_d(n)$ by $w(n)$ to convert the infinite duration impulse response the finite duration

$$h(n) = h_d(n)\, w(n)$$

- → The TF $H(z)$ of the filter is obtained by taking $ZT[h(n)]$.

Window Sequence for FIR filter design :-

Name of the window.	Window Sequence.
→ Rectangular window	$w_R(n) = \begin{cases} 1 ; & -\left(\frac{N-1}{2}\right) \le n \le \left[\frac{N-1}{2}\right. \\ 0 ; & \text{otherwise} \end{cases}$
→ Triangular window	$w_T(n) = \begin{cases} 1 - \frac{2\lvert n \rvert}{N-1} ; & -\frac{N-1}{2} \text{ to } \frac{N-1}{2} \\ 0 ; & \text{otherwise.} \end{cases}$

→ Hamming Window

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2n\pi}{N-1} ; & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 ; & \text{otherwise} \end{cases}$$

→ Hanning Window

$$w_{hn}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2n\pi}{N-1} & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 ; & \text{otherwise} \end{cases}$$

→ Blackman window

$$w_B(n) = \begin{cases} 0.42 + 0.5 \cos \frac{2n\pi}{N-1} + 0.08 \cos \frac{4n\pi}{N-1} ; & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 ; & \text{otherwise} \end{cases}$$

→ Kaiser window

$$w_K(n) = \frac{I_0\left(\alpha \sqrt{1 - \left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha)} \quad -\frac{N-1}{2} \le n \le \frac{N-1}{2}$$

P. Design an ideal highpass filter with frequency respo

$H_d(e^{j\omega}) = \begin{cases} 1 \; ; \frac{\pi}{4} \le |\omega| \le \pi \\ 0 \; ; |\omega| \le \frac{\pi}{4} \end{cases}$ find the value of $h(n)$ for

$n=11$ find $H(z)$ and plot the mag respons having

haning window.

Sol: step1:

$$h_d(n) = \frac{1}{2\pi} \int^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi} \left[\int_{\pi}^{-\pi/4} e^{j\omega n} d\omega + \int^{\pi/4} e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \int_{\pi}^{-\pi/4} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{j\omega n} d\omega.$$

$$h_{d(n)} = \frac{1}{n\pi} \left[\sin n\pi - \sin \frac{n\pi}{4} \right].$$

step2 Hamming window

$$w_{hn}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2n\pi}{N-1} \; ; & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 \; ; & \text{otherwise} \end{cases}$$

$$w_{hn}(n) \cdot \begin{cases} 0.5 + 0.5 \cos \frac{2n\pi}{5} \; ; & -5 \le n \le 5 \\ 0 \; ; & \text{otherwise} \end{cases} \quad \boxed{N=11}$$

step 3 $\boxed{h(n) = h_d(n) \cdot w_{hn}(n)}$ $-5 \le n \le 5$

$h_d(0) = 3/4 = 0.75$

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.075$

$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = 0.045$

$w_{hn}(0) = 1$

$w_{hn}(1) = w_{hn}(-1) = 0.5 + 0.5\cos \pi/5 = 0.9045$

$w_{hn}(2) = w_{hn}(-2) = 0.5 + 0.5\cos 2\pi/5 = 0.655$

$w_{hn}(3) = w_{hn}(-3) = 0.5 + 0.5\cos 3\pi/5 = 0.345$

$w_{hn}(4) = w_{hn}(-4) = 0.5 + 0.5\cos 4\pi/5 = 0.0945$

$w_{hn}(5) = w_{hn}(-5) = 0.5 + 0.5\cos 5\pi/5 = 0$

$h(n) = h_d(n) . w_{hn}(n)$

$h(0) = h_d(0) . w_{hn}(0) = 0.75 \times 1 = 0.75$

$h(1) = h(-1) = h_d(1)\, w_{hn}(1) = -0.225 \times 0.9045 = -0.204$

$h(2) = h(-2) = h_d(2)\, w_{hn}(2) = -0.159 \times 0.655 = -0.109$

$h(3) = h(-3) = h_d(3)\, w_{hn}(3) = -0.075 \times 0.345 = -0.026$

$h(4) = h(-4) = h_d(4)\, w_{hn}(4) = 0 \times 0.09 = 0$

$h(5) = h(-5) = h_d(5)\, w_{hn}(5) = 0.045 \times 0 = 0$

Step 4: TF of ~~the followin~~ filter given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)[z^{-n} + z^{n}]$$

$$H(z) = 0.75 + h(1)[z^{1} + z^{-1}] + h(2)[z^{2} + z^{-2}] + h(3)[z^{3} + z^{-3}]$$
$$+ h(4)[z^{4} + z^{-4}] + h(5)[z^{5} + z^{-5}]$$

$$= 0.75 - 0.204(z^{1} + z^{-1}) - 0.104(z^{2} + z^{-2}) - 0.026(z^{3} + z^{-3})$$

$$= 0.75 - 0.204z^{+1} - 0.204z^{-1} - 0.104z^{2} - 0.104z^{2} - 0.104z^{-2}$$
$$- 0.026z^{-3} - 0.026z^{3}$$

Step 5: T.F of Realizable filter is

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z) = z^{-5} H(z)$$

$$H'(z) = 0.75z^{-5} - 0.204z^{-6} - 0.204z^{-4} - 0.104z^{-7} - 0.104z^{-3}$$
$$- 0.026z^{-8} - 0.026z^{-2}$$

Step 6: Count filter coefficients

$$h(n) = h(N-1-n)$$

$H'(z) = -0.026 z^{-2} - 0.104 z^{-3} - 0.204 z^{-4} + 0.75 z^{-5} - 0.204 z^{-6}$

$- 0.104 z^{-7} - 0.026 z^{-8}$

$h(n) = h(N-1-n)$

$h(0) = h(10) = 0$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.026$

$h(3) = h(7) = -0.104$

$h(4) = h(6) = -0.204$

$h(5) = 0.75$

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos \omega n$$

$$= h(5) + \sum_{n=1}^{5} 2h(5-n) \cos \omega n$$

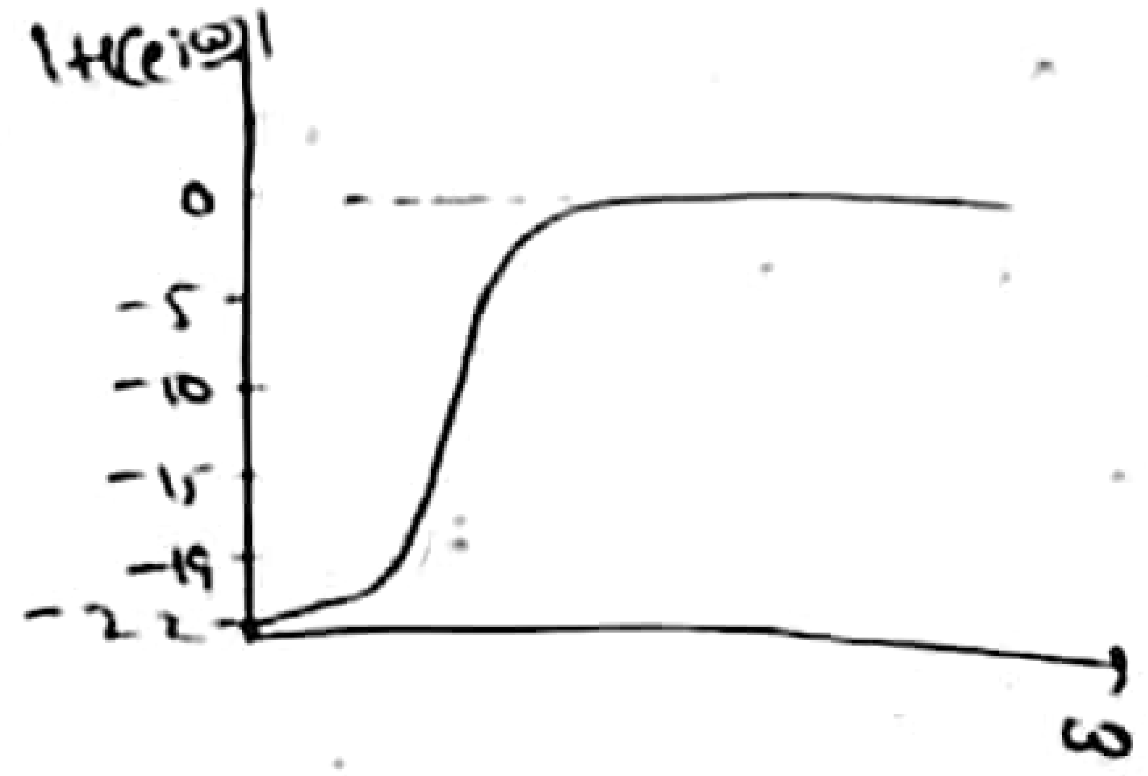
$$= h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega + 2h(1)\cos 4\omega$$

$$+ 2h(0)\cos 5\omega$$

$$= 0.75 + 2(-0.204)\cos\omega + 2(-0.104)\cos 2\omega + 2(-0.206)\cos 3\omega$$

$$|H(e^{j\omega})| = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$$

ω	0	15	30	45	60	75	90	105	120	135	150	165	180
$\lvert H(\omega)\rvert$ in dB	-21.72	-17.14	-10.67	-6.05	-3.07	-1.257	-0.3726	-0.0027	0.052	0.015	0	0	0.017



$h_d(0) = 0.75$

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.075$

$$\begin{cases} w_h(n) = 0.54 + 0.46\cos\left(\frac{2n\pi}{N-1}\right) \\ = 0, \text{ otherwise} \end{cases}$$

$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = 0.045$

$w_h(n) = w_h(-n)$

$w_h(0) = 0.54 + 0.46 = 1$

$w_h(1) = w_h(-1) = 0.54 + 0.46\cos(\pi/5) = 0.912$

$w_h(2) = w_h(-2) = 0.54 + 0.46\cos(2\pi/5) = 0.682$

$w_h(3) = w_h(-3) = 0.54 + 0.46\cos(3\pi/5) = 0.398$

$w_h(4) = w_h(-4) = 0.54 + 0.46\cos(4\pi/5) = 0.1678$

$w_h(5) = w_h(-5) = 0.54 + 0.46\cos(\pi) = 0.08$

The filter coefficients using hamming window sequence are

$h(n) = h_d(n) \cos w_H(n) \; ; \; -5 \le n \le 5$

$\quad 0 \qquad ; \text{otherwise}$

$h(0) = h_d(0) w_H(0) = 0.75 \times 1 = 0.75$

$h(1) = h(-1) = h_d(1) w_H(1) = -0.225 \times 0.912 = -0.2052$

$h(-2) = h(2) = h_d(2) w_H(2) = -0.159 \times 0.682 = -0.1084$

$h(-3) = h(3) = h_d(3) w_H(3) = -0.075 \times 0.398 = -0.03$

$h(-4) = h(4) = h_d(4) w_H(4) = 0$

$h(-5) = h(5) = h_d(5) w_H(5) = 0.045 \times 0.08 = 0.0036$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.75 + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{2} + z^{-3}\right]$$

$$+ h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right]$$

$$= 0.75 - 0.205z^{1} - 0.205z^{-1} - 0.108z^{2} - 0.108z^{-2} - 0.0323\,00$$

$$+ 0.0026z^{5} + 0.0026z^{-5}$$

Realizable T.F

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z) = z^{-5} H(z)$$

$$= 0.75z^{-5} - 0.205z^{-4} - 0.205z^{-6} - 0.108z^{-3} - 0.108z^{-7}$$
$$-0.03z^{-2} - 0.03z^{-8} + 0.0036z^{0} + 0.036z^{-10}$$

$$= 0.0036 - 0.03z^{-2} - 0.108z^{-3} - 0.205z^{-4} + 0.75z^{-5}$$
$$-0.205z^{-6} - 0.108z^{-7} - 0.03z^{-8} + 0.036z^{-10}$$

$h(n) = h(-n)$

$w_h(n) = w_h(-n)$

Causal filter coefficients.

$h(n) = h(N-1-n)$

$h(0) = h(10) = 0.0036$

$h(1) = h(9) = 0.$

$h(2) = h(8) = 0.08$

$h(3) = h(7) = -0.108$

$h(4) = h(6) = -0.205$

$h(5) = 0.75.$

Magnitude fu'n of $|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} h\left(\frac{N-1}{2}\right) \cos\omega n$

$$= h(5) + \sum_{n=1}^{5} 2h(5-n)\cos\omega n$$

$$= 0.75 + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega$$
$$+ 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.75 - 0.410\cos\omega - 0.28\cos 2\omega - 0.06\cos 3\omega + 0.007\cos$$

* The normalized Ideal (desired) freq. response for FI

filter design using windows.

→ Low pass filter

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\omega_c \le \omega \le \omega_c \\ 0 & ; -\pi \le \omega \le -\omega_c \\ 0 & , \omega_c \le \omega \le \pi \end{cases}$$

→ High pass filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_c \\ e^{-j\alpha\omega} & ; \omega_c \le \omega \le \pi \\ 0 & ; -\omega_c < \omega < \omega_c \end{cases}$$

→ Band pass filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\omega_{c2} \le \omega \le -\omega_{c1} \\ e^{-j\alpha\omega} & , \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & ; -\pi \le \omega < -\omega_{c2} \\ 0 & ; -\omega_{c1} < \omega < \omega_{c1} \\ 0 & ; \omega_{c2} < \omega \le \pi \end{cases}$$

→ Band stop filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_{c2} \\ e^{-j\alpha\omega} & ; -\omega_{c1} \le \omega \le \omega_{c1} \\ e^{-j\alpha\omega} & ; \omega_{c2} < \omega < \pi \\ 0 & ; -\omega_{c2} \le \omega \le -\omega_{c1} \\ 0 & , \omega_{c1} < \omega < \omega_{c2} \end{cases}$$

Ⓟ. Design a HPF using hamming window with a cut off freq. of 1.2 rad/sec and N=9.

Sol:- The desired freq. response $H_d(e^{j\omega})$ for a HPF is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega < \omega_c \\ e^{-j\alpha\omega} & ; \omega_c \le \omega \le \pi \\ 0 & ; -\omega_c < \omega < \omega_c \end{cases}$$

$N = 9.$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} H_d(e^{j\omega})\, e^{j\omega c} d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\alpha\omega}\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{n-\alpha}\right]_{-\pi}^{-\omega_c} + \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{n-\alpha}\right]_{\omega_c}^{\pi}$$

$$= \frac{1}{2\pi}\,\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{1}{2\pi}\,\frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}$$

$$= \frac{1}{2\pi j(n-\alpha)}\left\{-\left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right] + \left[e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}\right]\right\}$$

$$= \frac{1}{2\pi j(n-\alpha)}\left[-2j\sin\omega_c(n-\alpha) + 2j\sin\pi(n-\alpha)\right]$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left[-\sin\omega_c(n-\alpha) + \sin\pi(n-\alpha)\right] \quad , n \ne \alpha = \frac{N-1}{2}$$

for other values

$n = \alpha$. By using L-Hospital rule

$$h_d(n) = \lim_{n\to\alpha} \frac{1}{\pi(n-\alpha)}\left[\sin\omega_c(n-\alpha) + \sin\pi(n-\alpha)\right]$$

$$= \lim_{n\to\alpha} \frac{\sin\pi(n-\alpha)}{\pi(n-\alpha)} - \lim_{n\to\alpha} \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \Big]$$

$h_d(n) = 1 - \frac{\omega_c}{\pi}$. $\quad n = \alpha. = \frac{N-1}{2} = \frac{9-1}{2} = 4$.

Ⓟ. Design a band stop filter to reject freq. in the range 1 to 2 rad/sec. using rectangular window with $N=7$.

Sol:- $\omega_{c1} = 1$ rad/sec, $\omega_{c2} = 2$ rad/sec

$\alpha = \frac{N-1}{2} = 3$

The desired freq. response for BSF.

$$H(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_{c2},\ -\omega_{c1} \le \omega \le \omega_{c1},\ \omega_{c2} \le \omega \le \\ 0 & ;\ \text{other values} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\omega_{c2}} e^{-j\alpha\omega} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{-\omega_{c1}}^{\omega_{c1}} e^{-j\alpha\omega} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\omega_{c2}}^{\pi} e^{-j\alpha\omega} e^{j\omega n} d\omega$$

$$= \frac{1}{\pi(n-\alpha)} \Big[\sin(n-\alpha)\omega_{c1} + \underset{0}{\sin(n-\alpha)\pi} - \sin(n-\alpha)\omega_{c2} \Big].$$

At $n = \alpha$.

$h_d(n) = 1 + \frac{\omega_{c1}}{\pi} - \frac{\omega_{c2}}{\pi}$ for $h_d(3)$

Ⓟ. Design a Band pass FIR filter for the following specification Cutoff freq. $= 400$ Hz & 800 Hz. Sampling freq. 2000 Hz. $N = 11$.

Sol:- Cut off freq. $f_{c1} = 400$ Hz

$f_{c2} = 800$ Hz

$f_s = 2000$ Hz.

Normalized cutoff freq. $\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = \frac{2\pi \times}{}$

$$\omega_{c_2} = \frac{2\pi(800)}{\overset{10}{2000}} = \frac{4\pi}{5}$$

Frequency Sampling method

→ Design of FIR filter by freq. Sampling method.

procedure for Type - I

→ Choose the ideal (desired) freq. response $H_d(e^{j\omega})$

→ Sample $H_d(e^{j\omega})$ at N points by taking $\omega = \omega_k = \frac{2\pi k}{n}$

where $k = 0, 1, 2, \ldots N-1$. to generate the sequence

→ $\tilde{H}(k)$ (Tild).

$$\boxed{\therefore \tilde{H}(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} \quad \text{(or)} \quad H_d(\omega)}$$

→ Compute the N-sample of $h(n)$ using the following eqn.

Case I: N is odd

$$h(n) = \frac{1}{N}\left[\tilde{H}(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[\tilde{H}(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

Case II N is even

$$h(n) = \frac{1}{N}\left[\tilde{H}(0) + 2\sum_{k=0}^{\frac{N}{2}-1} Re\left[\tilde{H}(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

→ Take Z T of the impulse response of $h(n)$ to get the T.F $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}.$$

$$\boxed{\tilde{H}(k) = |H(k)| = |H(k)|\, e^{j\theta}}$$

for Linear phase filter $\theta = -\alpha\omega$

$$\theta = -\left(\frac{N-1}{2}\right)\frac{2\pi k}{N}$$

$$\theta = -\left(\frac{N-1}{N}\right)\pi k \; ; \quad k = 0, 1, \ldots N-1$$

The filter Coefficients h(n) Can be Obtained by finding IDFT of H(K)

$$h(n) = \frac{1}{N}\sum_{K=0}^{N-1} H(K)\, e^{\frac{j2\pi kn}{N}} \quad ; n = 0,1,2 \cdots N-1$$

If h(n) the impulse response of the filter is to be real valued sig the freq samples H(K) must satisfy the symmetry requirment for N odd or even

$$H(N-K) = H^*(K) \qquad ; K = 0,1,2 \cdots N-1$$

In addition for N even $h(\frac{N}{2}) = 0$. the magnetude respon with the freq response H(K) is an even function.

$$|H(K)| = |H(N-K)| \qquad ; K = 0,1, \cdots N-1.$$

The phase is an odd function : we represent phase fun

$$\theta(K) = -\theta(N-K)$$

$$\theta(N-K) = -\left(\frac{N-1}{N}\right)\pi(N-K)$$

$$= -\left(\frac{N-1}{N}\right)(\pi N - \pi K)$$

$$= -\left(\frac{N-1}{\cancel{N}}\right)\pi \cancel{N} + \left(\frac{N-1}{N}\right)\pi k$$

$$= -(N-1)\pi + \left(\frac{N-1}{N}\right)\pi k$$

θ(K) for N odd is given by θtk

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \; ; \; K = 0,1,2 \cdots N-1$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \; ; \text{for } K = \frac{N+1}{2} \cdots N$$

θ(K) for N even is given by

$$\theta(K) = -\left(\frac{N-1}{N}\right)\pi k \; ; \; n = 0,1, \cdots \frac{N}{2}-1$$

$$= 0 \quad ; \quad K = \frac{N}{2}$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \quad ; \quad K = \frac{N}{2} + 1 \quad ; \dots N-1$$

$$\theta(k) = -\theta(N-k)$$

for N odd

$$H(k) = |H(k)|\, e^{-j\left(\frac{N-1}{N}\right)\pi k} \quad \text{for } k = 0, 1, \dots \frac{N-1}{2}$$

$$= |H(k)|\, e^{+j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi k\right]} \quad \text{for } k = \frac{N+1}{2}, \dots N-1$$

for N even

$$H(k) = |H(k)|\, e^{-j\left(\frac{N-1}{N}\right)\pi k} \quad ; \quad k = 0, 1, \dots \frac{N}{2} - 1$$

$$= |H(k)| \quad ; \quad k = \frac{N}{2}$$

$$= |H(k)|\, e^{j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi k\right]} \quad ; \quad k = \frac{N}{2} + 1, \dots N-1$$

for N odd

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} \left[H(0) + 2 \sum_{k=1}^{N-1} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

for N even

$$h(n) = \frac{1}{N} \left[H(0) + 2 \sum_{k=1}^{\frac{N}{2}-1} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

Once the filter Coefficient $h(n)$ have been determine the sly fun filter is given by

$$\Rightarrow H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

P Determine the filter coefficients h(n) obtained by sampling $H_d(n) = H_d(e^{j\omega}) = e^{-j(N-1)\omega/2}$; $0 \le |\omega| \le \pi/2$

$= 0$; $\pi/2 \le |\omega| \le \pi$

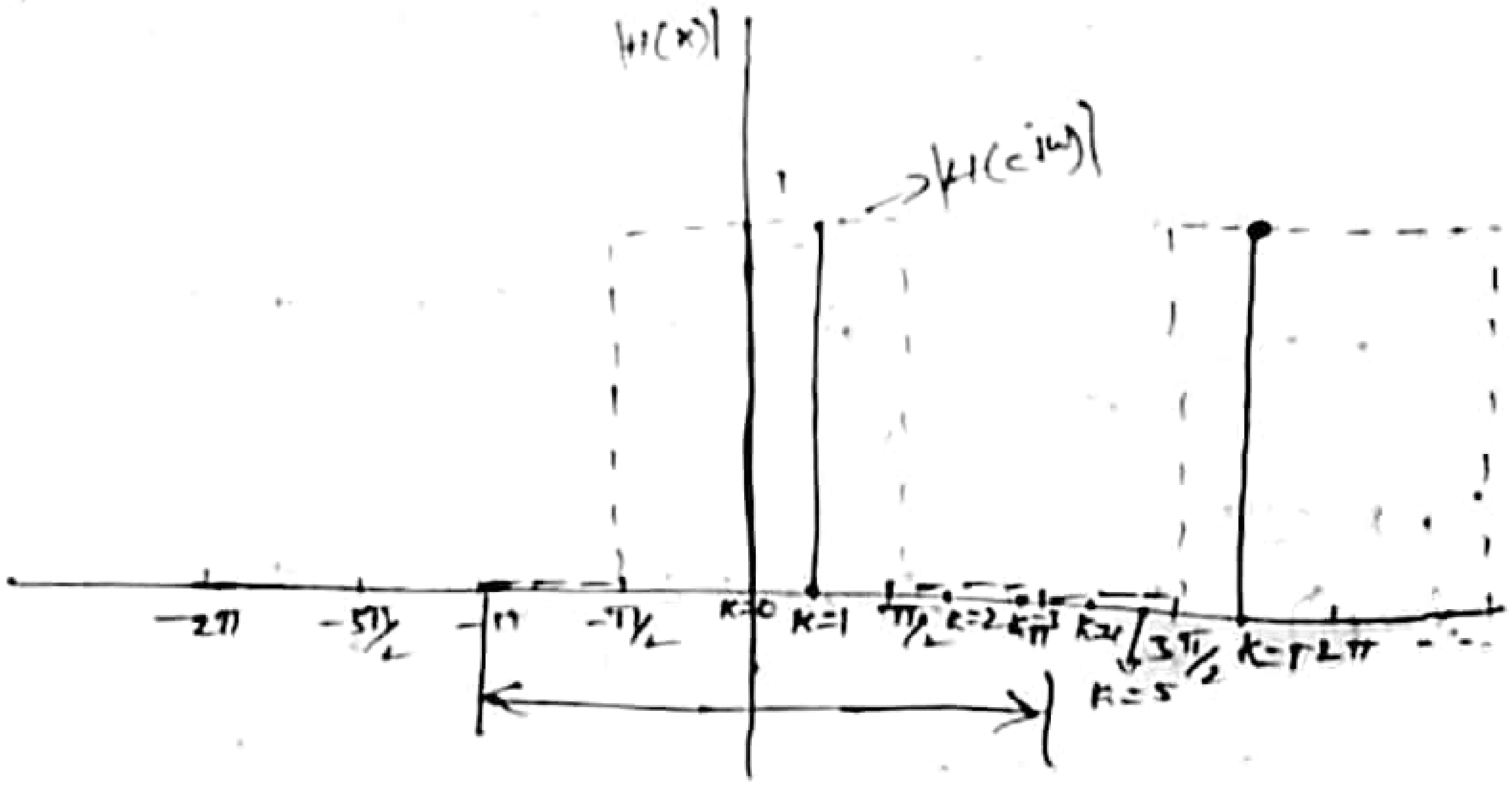
for N = 7.

Sol:- $\omega_k = \frac{2\pi}{N}k$, $k = 0, 1 \ldots N-1$.

$\tilde{H}(k) = H(k) = |H(k)| e^{j\theta(k)}$

$\omega_k = \frac{2\pi}{N}k$ $\quad k = 0, 1, 2, 3, 4, 5, 6$

$\omega_0 = 0$	$k = 0$	$\frac{2}{7} = 0.28$
$\omega_1 = \frac{2\pi}{7}$	$k = 1$	$\frac{4}{7} = 0.57$
$\omega_2 = \frac{4\pi}{7}$	$k = 2$	$\frac{6}{7} = 0.857$
$\omega_3 = \frac{6\pi}{7}$	$k = 3$	$\frac{8}{7} = 1.14$
$\omega_4 = \frac{8\pi}{7}$	$k = 4$	$\frac{10}{7} = 1.42$
$\omega_5 = \frac{10\pi}{7}$	$k = 5$	$\frac{12}{7} = 1.74.$
$\omega_6 = \frac{12\pi}{7}$	$k = 6$	



$|H(k)| = 1$; $k = 0, 1, 6$

$= 0$; $k = 2, 3, 4, 5$

Even

$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$; $k = 0, 1, 2, 3$

$= \frac{-6}{7}\pi k$; $k = 0, 1, 2, 3$

odd

$\theta(k) = (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k$

$= 6\pi - \frac{6\pi}{7}k$; $k = 4, 5, 6$

$H(k) = |H(k)| e^{j\theta(k)}$.

$H(k) = 1\, e^{-j6/7 \pi k} \quad ; k = 0,1$

$\quad\quad 0 \quad\quad , k = 2,3,4,5$

$\quad = 1\, e^{j[6\pi - \frac{6}{7}\pi k]} \quad ; \quad k = 6.$

N odd

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right\}\right]$$

$$= \frac{1}{7}\left[1 + 2\sum_{k=1}^{3} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right].$$

$$= \frac{1}{7}\left[1 + 2\ Re\left[e^{-j\frac{6}{7}\pi(1)}\, e^{\frac{j2\pi n(1)}{7}}\right]\right]$$

$$h(n) = \frac{1}{7}\left[1 + 2\ Re\left[e^{-j2\pi/7 (n-3)}\right]\right]$$

$$= \frac{1}{7}\left[1 + 2\cos\frac{2\pi}{7}(n-3)\right]$$

$h(n) = h(N-1-n)$

$h(n) = h(6-n)$

$h(0) = h(6) = 1/7\ \left(1 + 2\cos\frac{6\pi}{7}\right) = -0.11456.$

$h(1) = h(5) = 1/5\ \left(1 + 2\cos\frac{4\pi}{7}\right) = 0.099.$

$h(2) = h(4) = 1/7\ \left[1 + 2\cos\frac{2\pi}{7}\right) = 0.321$

$h(3) = h(3) = 1/7\ \left[1 + 2\cos(0)\right] = 0.42857.$

Using freq. Sampling method Design a BPF with the following specification.

→ Sampling freq, $f = 8000Hz$

→ cut off freq, $f_{c1} = 1000Hz$, $f_{c2} = 3000Hz$.

Determine the filter Coefficient $N=7$.

$\omega_k = \frac{2\pi}{N} k$

$\omega_0 = 0$, $k=0$ $\quad \omega_6 = \frac{12}{7}\pi \quad k=6$

$\omega_1 = \frac{2}{7}\pi$, $k=1$

$\omega_2 = \frac{4}{7}\pi$, $k=2$

$\omega_3 = 6/7\,\pi \quad k=3.$

$\omega_4 = 8/7\,\pi \quad k=4,$

$\omega_5 = 10/7\,\pi$, $k=5$

$$\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = \frac{2\pi\, 1000}{8000} = \pi/4.$$

$$\omega_{c2} = \frac{2\pi\, 3000}{8000} = 3\pi/4.$$

$|H(k)| = 1$; $k=1,2,5,6$

$\qquad 0$; $k=0,3,4.$

$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$; $k=0,1,2,3$

$\qquad = -6/7\,\pi k$; $k=0,1,2,3$

$\theta(k) = (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k$; $k=4,5,6$

$\qquad = 6\pi - 6/7\,\pi k$; $k=4,5,6.$

$H(k) = |H(k)|\, e^{j\theta(k)}$

$H(k) = 0$; $k=0,3,4.$

$\qquad = 1\; e^{-j6/7\,\pi k}$; $k=1,2$

$\qquad = 1\; e^{j(6\pi - 6/7\,\pi k)}$; $k=5,6.$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(k)\, e^{j\frac{2\pi n k}{N}}\right)\right]$$

$$= \frac{1}{7}\left[0 + 2\sum_{k=1}^{3} \mathrm{Re}\left[H(k)\, e^{j\frac{2\pi n k}{N}}\right)\right]$$

$$= 1/7\left[-\mathrm{Re}\left[e^{-j\,6/7\,\pi}\, e^{j\frac{2\pi n}{7}}\right)\right] + 2\,\mathrm{Re}\left[e^{-j\,12\pi/7}\, e^{j4\pi}\right.$$

$$= \frac{1}{7}\left[2\,\mathrm{Re}\left[e^{\frac{j2\pi}{7}(n-3)}\right] + 2\,\mathrm{Re}\left[e^{j4\pi/7\,(n-3)}\right]\right]$$

$$h(n) = \frac{1}{7}\left[2\cos 2\pi/7\,(n-3) + 2\cos\frac{4\pi}{7}(n-3)\right)$$

$$h(n) = h(N-1-n)$$

$$h(0) = h(6) = \frac{1}{7}\left[2\cos 6\pi/7 + 2\cos 12/7\,\pi\right) = -0.0992$$

$$h(1) = h(5) = \frac{1}{7}\left[2\cos 4\pi/7 + 2\cos\frac{8}{7\pi}\right) = -0.321$$

$$h(2) = h(4) = \frac{1}{7}\left(2\cos\frac{2\pi}{7} + 2\cos 4\pi/7\right) = 0.1145$$

$$h(3) = \frac{1}{7}\left[2\cos 0 + 2\cos 0\right) = 4/7 = 0.57$$

Structures for realization of FIR ~~filter~~ Sly's :-

The diff. types of structures for realizing the FIR Sly are

1. direct form realization.
2. Transposed form realization.
3. Cascade realization
4. Lattice structure realization,
5. Linear phase realization

The T.F of FIR sly is

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{K=0}^{N-1} b_K z^{-K}$$

$$= b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots b_{N-1} z^{-(N-1)}$$

$$H(z) = Z.T[h(n)] = \sum_{n=0}^{N-1} h(n) z^{-n}$$

change index in to k

$$= \sum_{K=0}^{N-1} h(K) z^{-K} = h(0) + h(1) z^{-1} + \dots h(N-1)z^{-(N-1)}$$

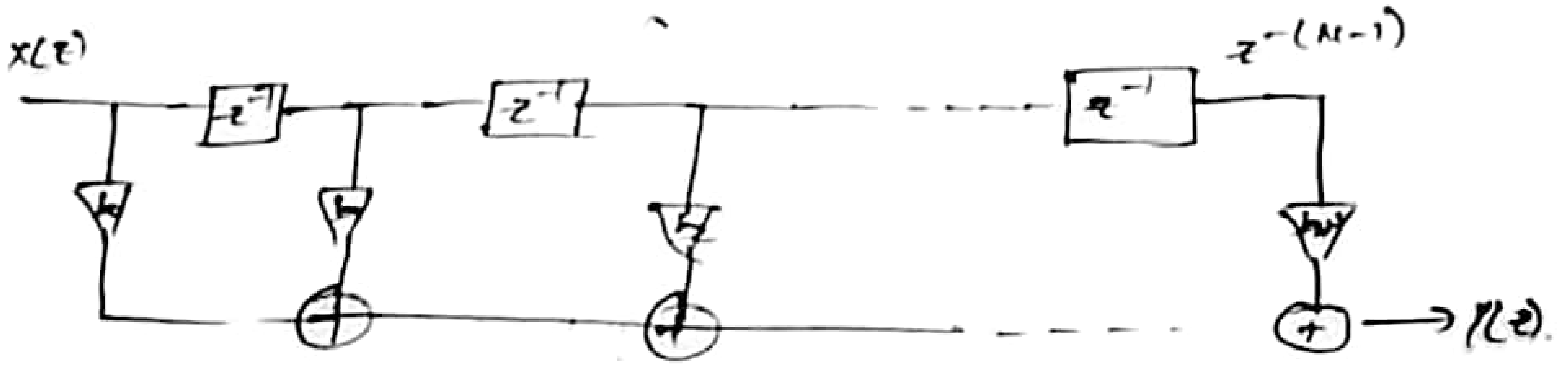
$b_0 = h(0), \; b_1 = h(1) \dots b_K = h(K)$

Direct form realization of FIR sly :-

The direct form structure can be obtained from the general eq'n Y(z) of the FIR sly

$$Y(z) = \sum_{K=0}^{N-1} b_K z^{-K} X(z)$$

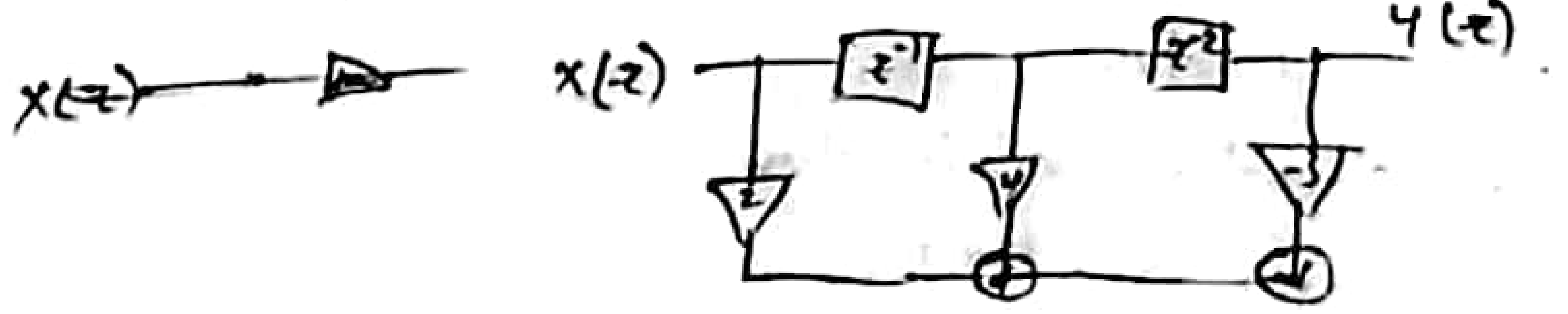
$$= b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \dots b_{N-1} z^{-(N-1)} X(z)$$

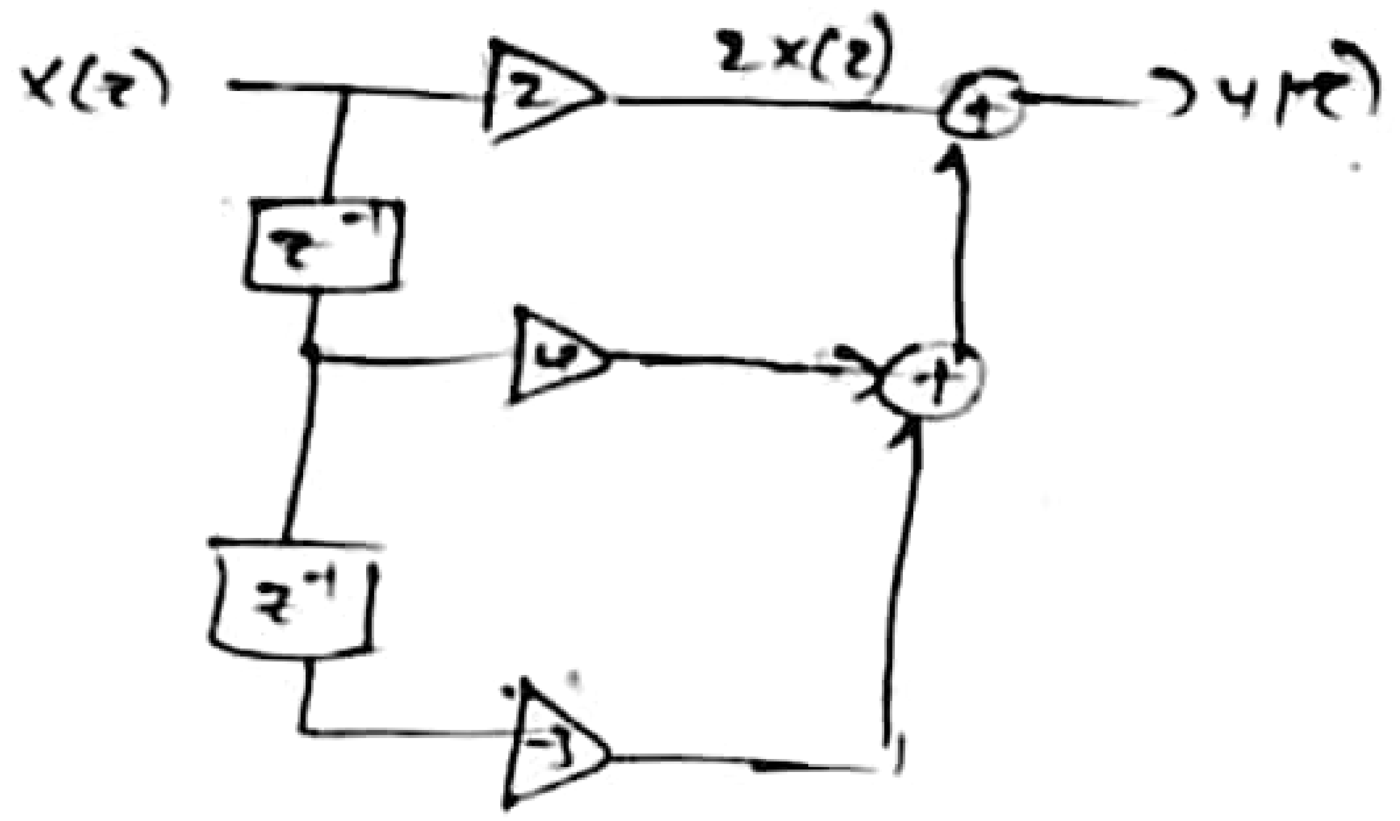


Ⓟ. Realize the second order FIR sly $y(n) = 2x(n) + 4x(n-1) - 3x(n-2)$. using direct form structure.

Sol:- Given $y(n) = 2x(n) + 4x(n-1) - 3x(n-2)$

$$Y(z) = 2X(z) + 4z^{-1}X(z) - 3z^{-2}X(z)$$





Lattice structure Realization of FIR Systems:-

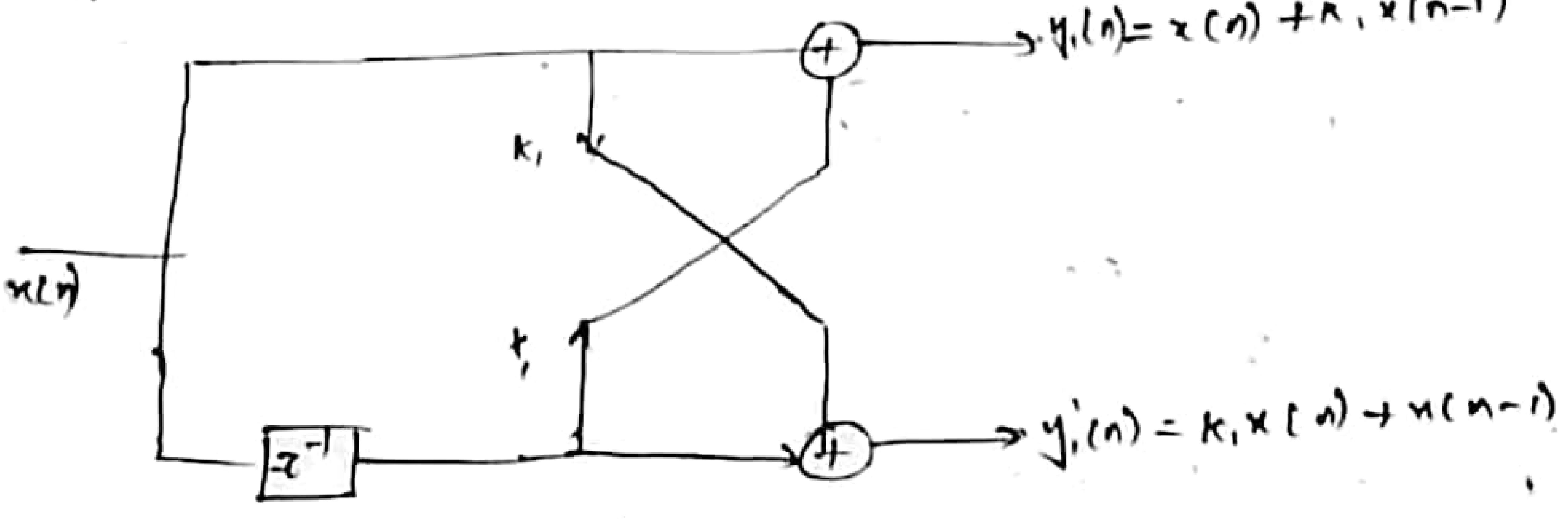
The lattice structure Consists of two diff. paths through which the i/p x(n) is process. Hence the lattice structure has two diff. output setups y(n) & y'(n).

y(n) is the real o/p & y'(n) is the supporting o/p. which offers supports for obtaining the o/p for the next stage.

A single stage lattice structure shown in the following fig. where in k is reflection Coefficient.

$y_1(n) = x(n) + k_1 x(n-1)$

$y_1'(n) = k_1 x(n) + x(n-1)$.



(P). ~~Realize the s/y~~

Procedure to realize the Lattice structure of the FIR s/y:-

→ If the Coeff of the present i/p x(n) is not unity. Convert it to unity by taking Common of the Coefficient of the present i/p

→ Find the order of the diff. eq'n and Compare the Coeff. of the given diff eq'n with the Coeffs. of the same order Lattice structure O/p involving the reflection Coeff. $k_1, k_2, k_3 \ldots$

→ Assign the Calculated values of '$k_1, k_2 \ldots$ and Constr the structure.

Ⓟ. Realize a sly with G $H(z) = 5+3z^{-1}$ by using Latti structure.

Sol:- Given $H(z) = 5+3z^{-1}$

$$\frac{Y(z)}{X(z)} = 5+3z^{-1}$$

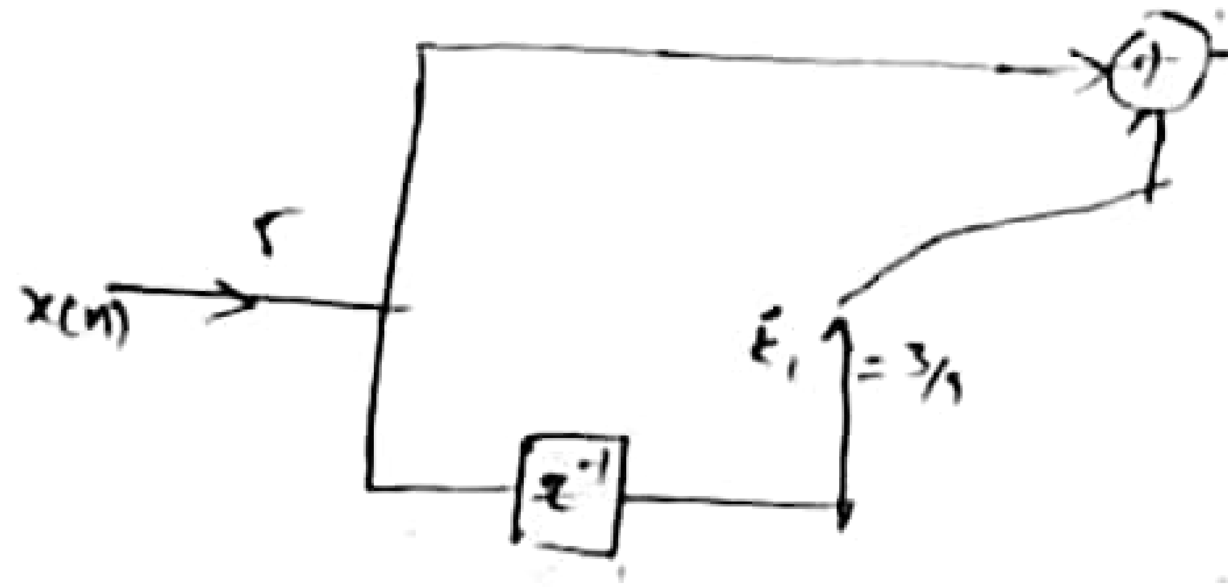
$$Y(z) = 5X(z) + 3z^{-1}X(z)$$

$$y(n) = 5x(n) + 3x(n-1)$$

$$= 5\left[x(n) + \frac{3}{5}x(n-1)\right]$$

$$y_1(n) = x(n) + k_1 x(n-1)$$

$$k_1 = 3/5$$



$$\rightarrow y(n) = 5\left[x(n) + \tfrac{3}{5}x(n-1)\right].$$

Two stage lattice structure :—

The output from the second stage of the Lattice structure which is the second order FIR sly is given as follows.

$$y_2(n) = y_1(n) + k_2 y_1'(n-1)$$

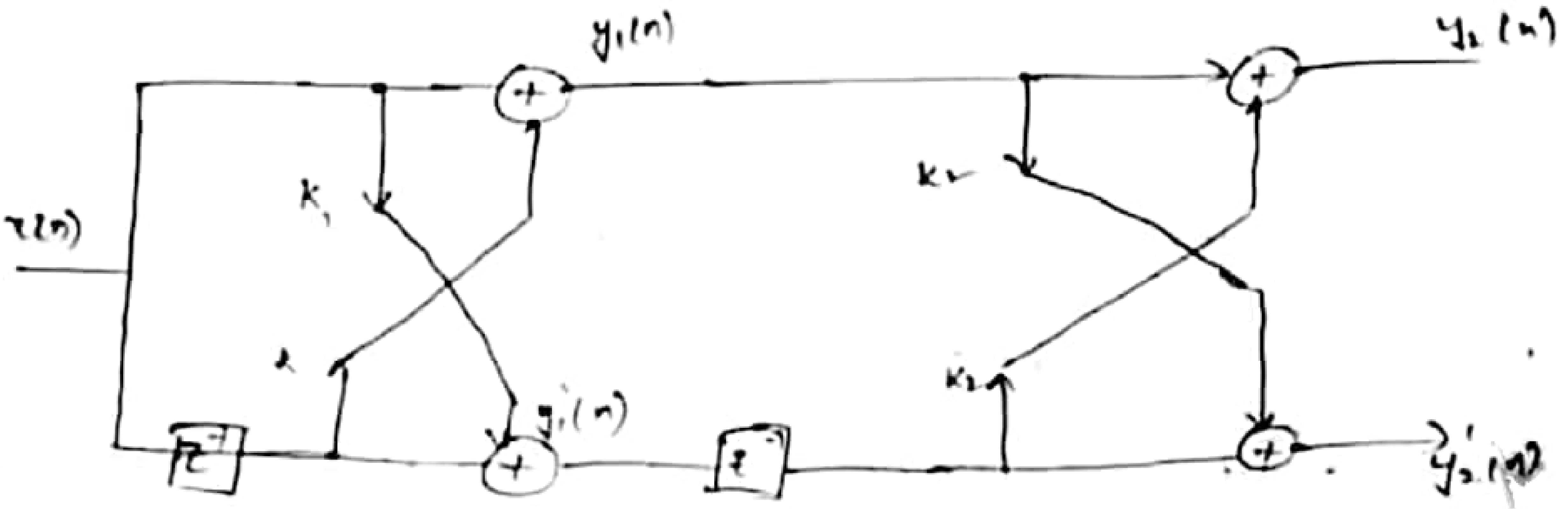
$$y_2'(n) = k_2 y_1(n) + y_1'(n-1)$$

$y_2(n) = [x(n) + k_1 x(n-1)] + k_2 [k_1 x(n-1) + x(n-2)]$

$= x(n) + k_1 x(n-1) [1 + k_2] + k_2 \, x(n-2)$

$y_2'(n) = k_2 [x(n) + k_1 x(n-1)] + [k_1 x(n-1) + x(n-2)]$.

$= k_2 x(n) + k_1 x(n-1) [1 + k_2] + x(n-2)$



① Determine the lattice coeff. Corresponding to the FIR Sly with the sly fcn $H(z) = 1 + 7/9 \, z^{-1} + 3/5 \, z^{-2}$ and realize it.

Sol:- $H(z) = 1 + 7/9 \, z^{-1} + 3/5 \, z^{-2}$

$Y(z) = X(-z) + 7/9 \, z^{-1} X(z) + 3/5 \, z^{-2} X(z)$

$y(n) = x(n) + 7/9 \, x(n-1) + 3/5 \, x(n-2)$ —— ②

6/3/19

UNIT-VI

Introduction To programmable DSP

Architecture features of DSP processors:

1) DSP processor should have multiple Register so that data can exchanged from Rigister to Register fast. 2) DSP operations required. multiple operands simultaneously. Hence DSP processor should have multiple operand fetching capacitor.

3) DSP should have buffers to support the shifting operations.

4) The DSP processor should able to perform multiple/and accumulate operations very fast.

5) DSP processor should have multiple pointers to support multiple operands, jumps and shifts operation

6) DSP processor can be used with generator processor they should have multi processing capacity

7) To suppourt DSP operations DSP processor should have on chip memory

8) For real time Applications interrupts and timers are required. Hence DSP process should have powerfull interrupts and timers.

7/3/19

MAC (Multiply and Accumulator):

most of the operations in DSP involue array multiplication the operations such as convolution, correlation require mutiply and accumulate operations.

In real time applications the array multiplication and accumulation must be completed before next sample of i/p comes.

This requires very fast implementation of multiplication and accumulation.

For this purpose the dedicated H/W unit MAC is used is called multiplier and accumulator.

The complete MAC operation is executed in one clock cycle.

In the DSP processor the o/p of the multiplier is store into the product register this product register containg are added to the Accumulator Register the central ALU.

The DSP process have the special instruction called MACD. ie; multiply and accumulate with data shift.

MAM (Multiple Acesses Memory)

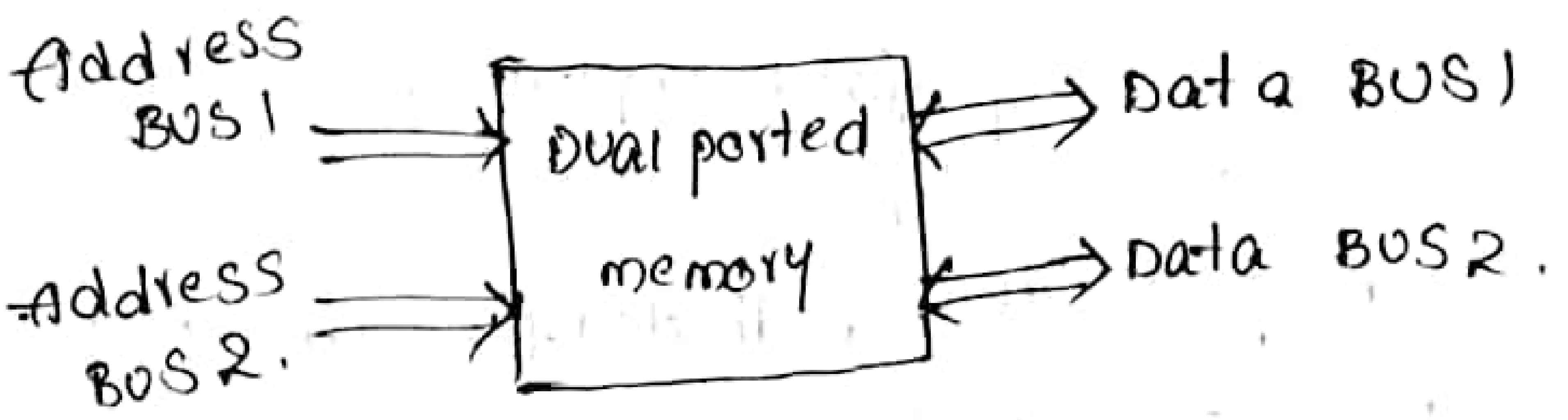
The MAM allows more than one memory Accesses in a single clock cycle. the Dual Acesses RAM. (DA-RAM) allows 2-memory Accesses in a single clock cycle.

The Dual Acesses RAM is connected to the DSP processes with 2 Address. with 2 data buses independently. this gives 4 memory Accesses in a single clock period.

MPM (Multi ported Memory):

The MPM has the facility of interfacing multiple Address and data buses. the following fig shows the Dual ported memory. This memory has 2 Address buses and 2 data buses saperatly interfaced.

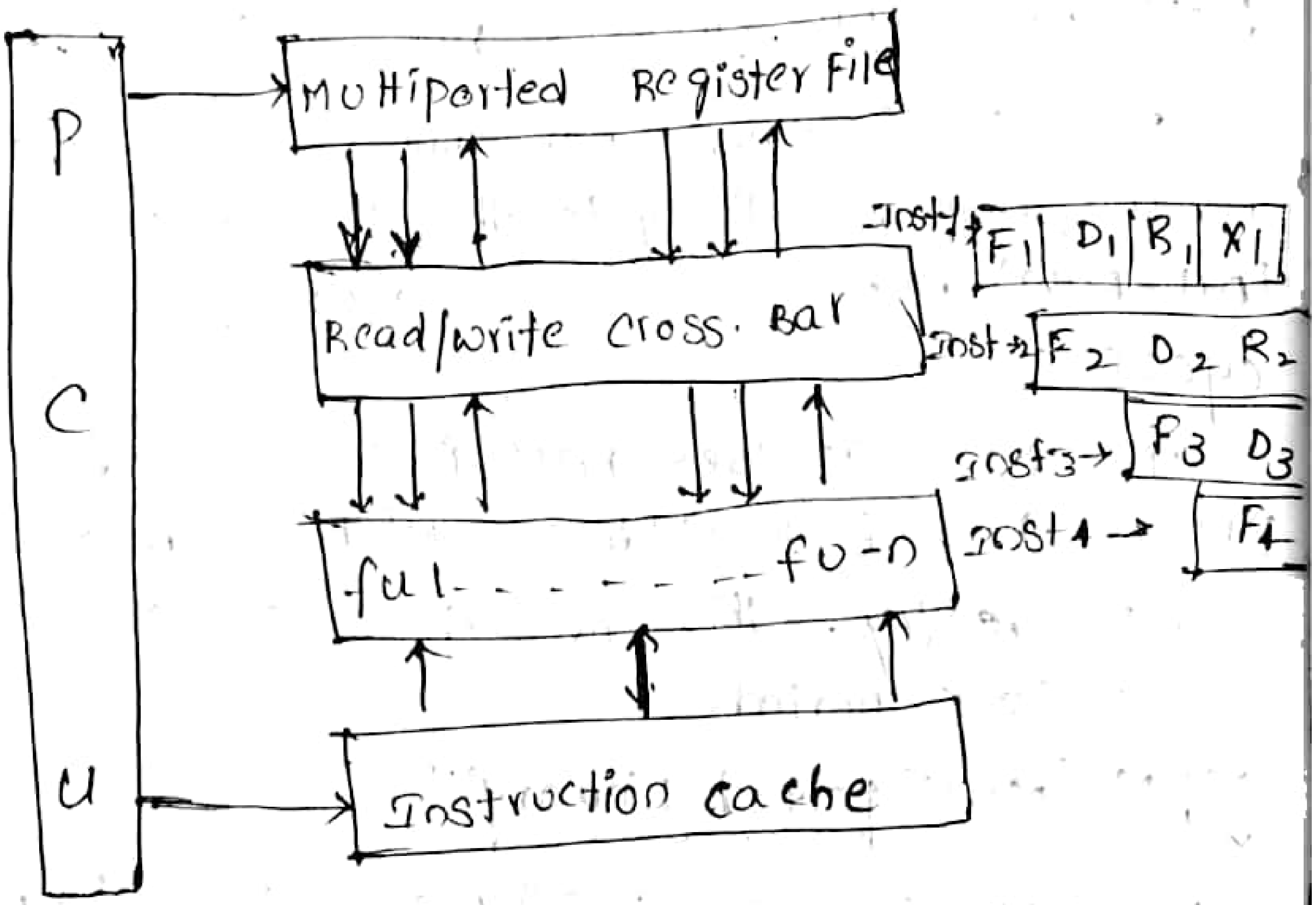
The Dual ported memory can allows 2 memory Accesses in a single clock period.



With the help of dual port memory the program and data can be store in a single memory chip and they can be acessed simultaneously.

The multi ported memories having increased no. of pins larger chip area which makes it more expensive and large in size.

VLIW (very long Instruction word):



PCU → program control unit.

fu → function unit.

9/3/19. Sum of the DSP process use ~~ver~~ VLIW architecture such architecture consists of multiple no. of ALU's, MAP units and shifter Registers.

The following fig shows the block diag of VLIW architecture.

The above Architecture consist of multi ported register file. it is used for fetching the operands and storing the results.

The Read/write cross bar provides parallel Random acesses by functional units to the multiported Register file.

The function units work concurrently with the load (or) store operation of the data b/w RAM and Register file.

The program control unit provides the algorithm that executes independent ||el operations.

The performance of VLIW Architecture depends on degree of parallelism.

Normally 8-functional units are preferred this number is limed by H/W cost of the multiported. Register file and Read/write cross bar.

Pipelining:

without pipelining

Value of T	Fetch	Decode	Read	execute.
1	I_1			
2	~~I_2~~	I_1		
3		~~I_2~~	I_1	
4			~~I_2~~	I_1
5	I_2			~~I_2~~
6		I_2		
7			I_2	
8.				I_2

With pipelining :

Value of T	fetch	Decode	Read	Execute.
1	I_1			
2	I_2	I_1		
3	I_3	I_2	I_1	
4	I_4	I_3	I_2	I_1
5	I_5	I_4	I_3	I_2
6		I_5	I_4	I_3
7			I_5	I_4
8				I_5.

9/3/19.

Any instruction cycle can be split into the following instruction.

1) fetch : In this phase an instruction is fetch from the memory.

2) Decode : In this phase an instruction is decoded.

3) Read : An operand required for this instruction is fetch from the data memory.

4) execute : The operation is executed and results are stored at Appropriate place.

Each of the above instructions can be saperately executed in different functional units. to the following fig shows 1) How the instruction is executed without pipeline.

2) from this fig. we can observe that when the instruction I_1 is in fetch, the other units such as

decode, Read and Execute are ideal. this means each functional unit is busy only for 25% of the total time.

from the fig (2) shows the instruction execution with pipeline,

Here we can observe that instruction-1 is in decode phase. next instruction I_2 is fetched.

Similarly, when I_2 goes to decode phase, next instruction I_3 is fetched.

∴ we observed that all the functional units are executed 4 successful instructions at any time.

compare above two figs we can observed that 5 instructions are executed. the same time pipelining misused.

Special Addressing modes:

The conventional microprocess have addressing modes such as direct, indirect, immediate Addressing modes. The DSP process have additional addressing modes because of fetch execution is fast.

1) Short immediate Addressing mode:

The operand is specified using a short constant this short constant becomes a part of a single word instruction.

In DSP process 8-bit operand can be specified. as one of the operand. in single word instructions such as. add, subtract, Div, AND, OR, NAND.. etc.

2) Short direct Addressing mode:

The lower order Address of the operand is specified in the single word instruction in DSP process the lower 7-bits of the Address are specified as a part of instruction.

Higher 9-bits of the Address are stored the data

Phase point.

3) Memory Map Addressing mode:.

The cpu and I/O registers are accessed are memory location this registers are maped in the starting page (or) ending page of the memory phase

The page-0 corresponds to the starting page of the memory space

4) Indirect addressing mode:.

The addressing of the operands are stored in the Indirect Address registers.

In DSP process such registers are called Auxiliary registers.

Any of these auxiliary registers can be updated in the operands fetched by these registers are being executed.

The auxiliary registers are incremented (or) decremented Automatically by the value specified. In offset Registers. these offset registers called index registers

5) Bit Reversal Addressing modes:.

for the calculation of FFT the i/p data is required in bit-Reversal order. there is no need to re-shuffle the data in bit reversal sequence

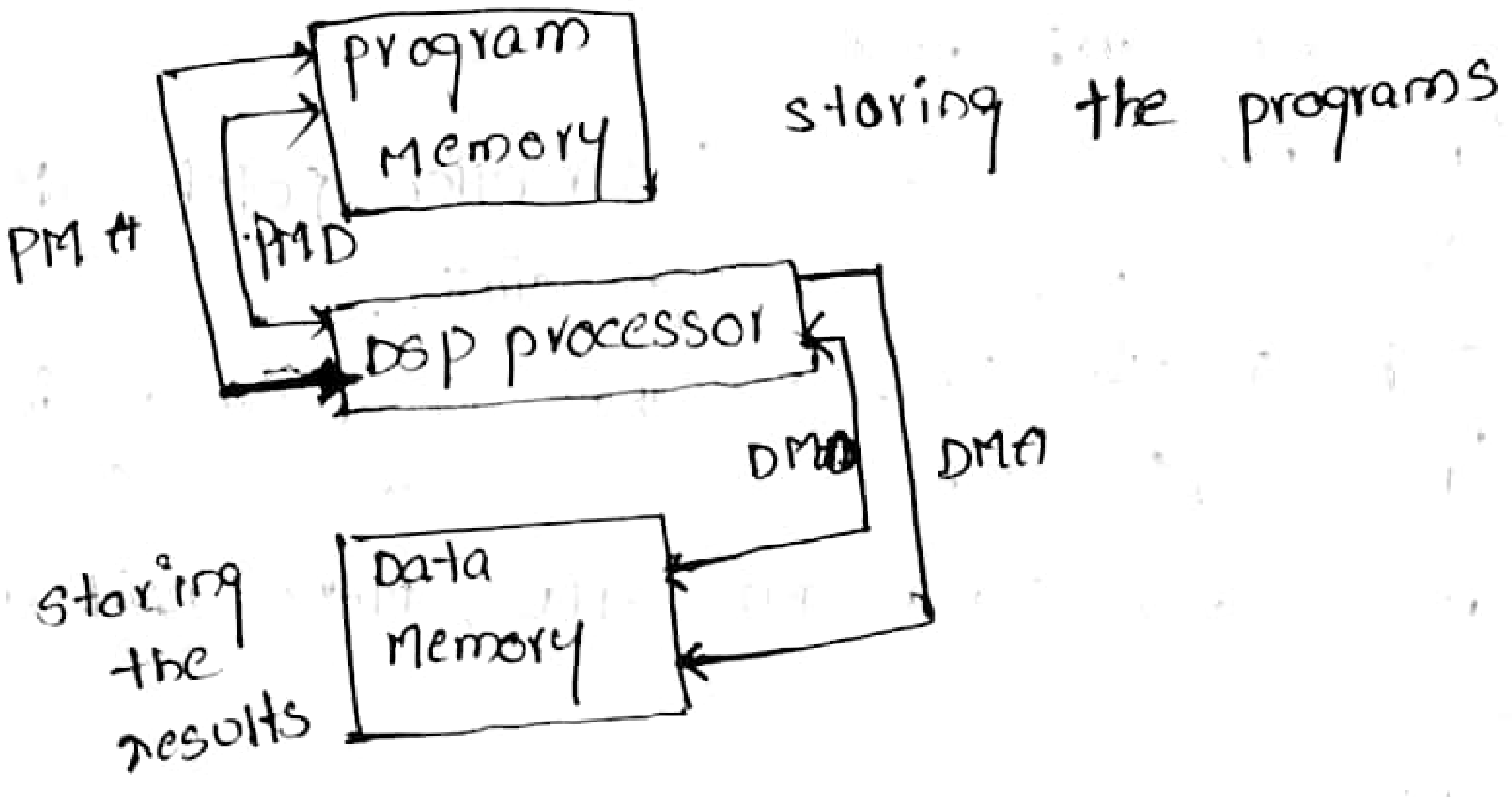
The serially arranged the data in the memory given to the processor in bit reversal mode. with the help of Bit - reversal addressing mode

6) Circular Addressing mode:.

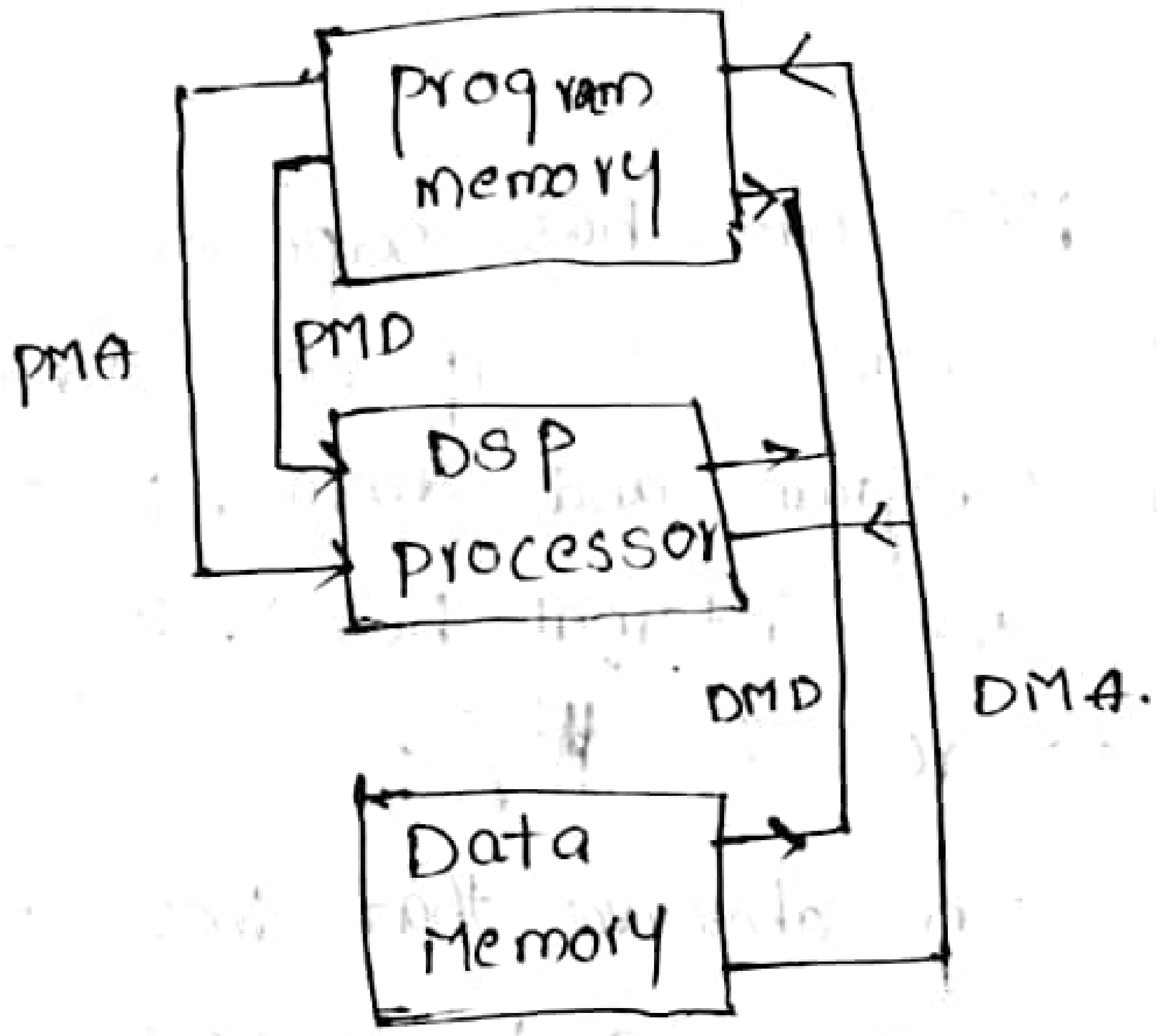
If this mode the data stored in the memory can be read (or) write in circular fashion.

The memory is organized as a circular buffer the begining and ending addresses of the circular buffer are contineoulsly monitor.

12/3/19 ***Harvard Architecture:-**



Modified Harvard Architecture:-



MACD Instruction performs multiply and accumulate in the memory accesses are required.

1) Fetch MACD Instruction from program memory.
2) fetch one of the operands. from program memory.
3) Fetch second operand from data memory.
4) Data Memory write.

If this instruction is executed with von neuman Architecture. It requires 4 block cycles but Harvard and modified Harvard requires less no. of clock cycles.

vonneuman Architecture:.

The General purpose process normally we have this type of Architecture the Architecture shapes same memory for program and data.

The process perform instruction fetch, decode, Read and execute operations sequencially.

such Architecture in the speed can be increased by pipelining this type of Architecture contains common Address and Data BUS, ALU, MAC unit and I/o. devices.

This type of Architecture is not suitable for DSP processes.

Harvard Architecture:

the Harvard Architecture has saperate memory for program and data there also saperate Address and data buses for program and data, because of these saperate on chip memory the internal buses. the speed of execution Architecture is High.

from the fig we can observe that there is PMA BUS [program memory Address] and PMD [program memory Data] BUS saperate from program.

Similarly, there is saperate data memory Data (DMD) BUS and data memory Address [DMA] BUS.

The DSP includes various Registers, Address generators and ALU...

The PMD BUS is used to get instructions from the program memory and DMD is used to exchange operands and results from Data memory.

The Instruction code from program memory and operands from data memory can be search simoultaneously is parallel operation increases the speed.

Modified Harvard Architecture:

From this Architecture one set of bus to access program as well as data memory.

The DMD bus can be use to transfer the data from program memory to data memory. and vice-versa. Normally, the program memory and data memory address on generated by the saperated address generators, this modified Harvard Architecture. used in several programmable DSP.

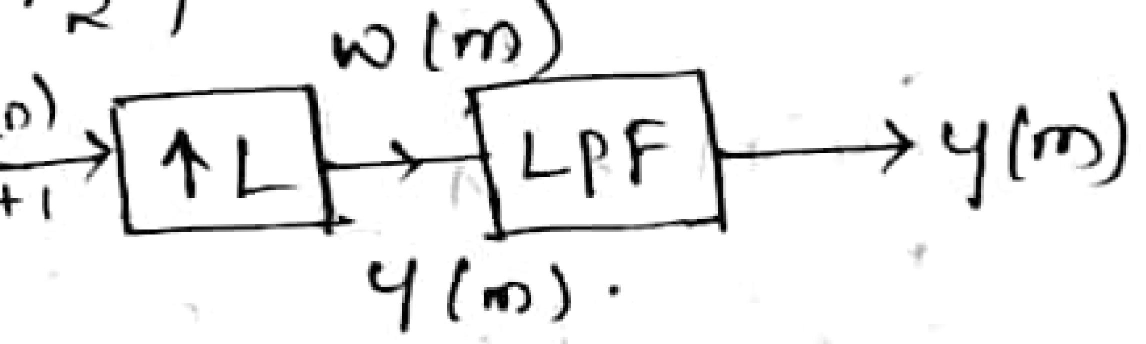
14/3/19 ** 9) The sequence $x(n) = (0, 2, 4, 6, 8)$ is interpolated using interpolation sequence $b_K = [1/2, 1, 1/2]$ and the interpolation factor is '2' to find the interpolated sequence $y(m)$. the 8

Sol) Given sequence $x(n) = [0, 2, 4, 6, 8)$.

$b_K = (1/2, 1, 1/2)$

$L = 2$. -1 $\overset{\uparrow}{0}$ $+1$ $x(0)$

$(L-1)$ zero's.



$w(m) = [\underset{0}{0}, \underset{1}{0}, \underset{2}{2}, \underset{3}{0}, \underset{4}{4}, \underset{5}{0}, \underset{6}{6}, \underset{7}{0}, \underset{8}{8}]$.

$y(m) = \Sigma\, b_K\, w(m-k)$.

$y(m) = \Sigma\, b_K\, w(m+1) + b_K\, w(m-0) + b_K\, w(m-1)$

$y(0) = b_{-1}\, w(0+1) + b_0\, w(0-0) + b_1\, w(0-1)$

$y(0) = 1/2\,(0) + 1\,(0) + 1/2\,(0)$

$y(0) = 0$

$y(1) = b_K\, w(1+1) + b_K\, w(1-0) + b_K\, w(1-1)$

$y(1) = b_{-1}\, w(2) + b_0\, w(1) + b_1\, w(0)$.

$$= \tfrac{1}{2}(2) + 1(0) + \tfrac{1}{2}(0)$$

$$\boxed{y(1) = 1}$$

$$y(2) = b_K\, w(2+1) + b_K\, w(2-0) + b_K\, w(2-1)$$

$$= b_{-1}\, w(3) + b_K\, w(2) + b_K\, w(1)$$

$$= \tfrac{1}{2}(0) + 1(2) + \tfrac{1}{2}(0)$$

$$\boxed{y(2) = 2}$$

$$y(3) = b_K\, w(3+1) + b_K\, w(3-0) + b_K\, w(3-1)$$

$$= b_{-K}\, w(4) + b_0\, w(3) + b_{+1}\, w(2)$$

$$= \tfrac{1}{2}(4)^2 + 1(0) + \tfrac{1}{2}(2)$$

$$\boxed{y(3) = 2+1} \Rightarrow \boxed{y(3) = 3}$$

$$y(4) = b_K\, w(4+1) + b_K\, w(4-0) + b_K\, w(4-1)$$

$$= b_{-1}\, w(5) + b_0\, w(4) + b_{+1}\, w(3)$$

$$= \tfrac{1}{2}(0) + 1(4) + \tfrac{1}{2}(0)$$

$$\boxed{y(4) = 4}$$

$$y(5) = b_K\, w(5+1) + b_K\, w(5-0) + b_K\, w(5-1)$$

$$= b_{-1}\, w(6) + b_0\, w(5) + b_{+1}\, w(4)$$

$$= \tfrac{1}{2}(6)^3 + 1(0) + \tfrac{1}{2}(4)^2$$

$$y(5) = 3+2$$

$$\boxed{y(5) = 5}$$

$$y(6) = b_K\, w(6+1) + b_K\, w(6-0) + b_K\, w(6-1)$$

$$= b_{-1}\, w(7) + b_0\, w(6) + b_{+1}\, w(5)$$

$$= \tfrac{1}{2}(0) + b_0(6) + \tfrac{1}{2}(0)$$

$$\boxed{y(6) = 6}$$

$y(7) = b_K \; w(7+1) + b_K \; w(7-0) + b_K \; w(7-1)$

$= b_{-1} \; w(8) + b_0 \; w(7) + b_{+1} \; w(6)$

$= \frac{1}{\cancel{2}} (\cancel{8})^{4} + 1 \; (0) + \frac{1}{\cancel{2}} (\cancel{6})^{3}$

$y(7) = 4 + 3$

$\boxed{y(7) = 7}$

$y(8) = b_K \; w(8+1) + b_K \; w(8-0) + b_K \; w(8-1)$

$= b_{-1} \; w(9) + b_0 \; w(8) + b_{+1} \; w(7)$

$= \frac{1}{2} \; (0) + 1 \; (8) + \frac{1}{2} \; (0)$

$\boxed{y(8) = 8}$

$y(m) = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ ✓

2) The sequence $x(n) = [0, 3, 6, 9]$ is interpolated using interpolation sequence $b_K = [1/3, 2/3, 1, 2/3, 1/3]$. and the Interpolation factor is "3" find the Interpolated sequence $y(m)$.

sol) Given data.

$x(n) = [0, 3, 6, 9)$

$b_K = [1/3, 2/3, 1, 2/3, 1/3)$

$\quad\quad\; -2 \quad -1 \quad \underset{0}{\uparrow} \quad +1 \quad +2.$

$L - 1 = 0$

$3 - 1 \Rightarrow 2.$

$w(m) = \{0, 0, 0, 3, 0, 0, 6, 0, 0, 9\}$

$y(m) = \Sigma \; b_K \; w(m-K)$

$y(m) = b_K \; w(m+2) + b_K \; w(m+1) + b_K \; w(m-0) + b_K \; w(m+1) + b_K \; (m-2).$

$y(0) = b_{-2} \; w(2) + b_{-1} \; w(1) + b_0 \; w(0) + b_1 \; w(-1) + b_2 \; (-2)$

$= \frac{1}{3}(0) + \frac{2}{3}(0) + 1(0) + \frac{2}{3}(0) + \frac{1}{3}(0)$

$\boxed{y(0) = 0}$

$y(1) = b_{-2}\, w(3) + b_{-1}\, w(2) + b_0\, w(1) + b_1\, w(0) + b_2\, w($

$= \frac{1}{\cancel{3}}(\cancel{3}) + \frac{2}{3}(0) + 1(0) + \frac{2}{3}(0) + \frac{1}{3}(0)$

$\boxed{y(1) = 1}$

$y(2) = b_{-2}\, w(4) + b_{-1}\, w(3) + b_0\, w(2) + b_1^{w}(1) + b_2^{w}(0)$

$= \frac{1}{3}(0) + \frac{2}{\cancel{3}}(\cancel{3}) + 1(0) + \frac{2}{3}(0) + \frac{1}{3}(0)$

$\boxed{y(2) = 2}$

$y(3) = b_{-2}\, w(5) + b_{-1}\, w(4) + b_0\, w(3) + b_1\, w(2)$
$+ b_2\, w(1)$

$= \frac{1}{3}(0) + \frac{2}{3}(0) + 1(3) + \frac{2}{3}(0) + \frac{1}{3}(0)$

$\boxed{y(3) = 3}$

$y(4) = b_{-2}\, w(6) + b_{-1}\, w(5) + b_0\, w(4) + b_1\, w(3)$
$+ b_2\, w(2)$

$= \frac{1}{\cancel{3}}(\overset{2}{\cancel{6}}) + \frac{2}{3}(0) + \overset{1}{\cancel{b_0}}(0) + \frac{2}{\cancel{3}}(\cancel{3}) + \frac{1}{3}(0)$

$= 2 + 2.$

$\boxed{y(4) = 4}$

$y(5) = b_{-2}\, w(7) + b_{-1}\, w(6) + b_0\, w(5) + b_1\, w(4)$
$+ b_2\, w(3)$

$= \frac{1}{3}(0) + \frac{2}{\cancel{3}}(\cancel{6})^2 + 1(0) + \frac{2}{3}(0) + \frac{1}{\cancel{3}}(\cancel{3})$

$= 4 + 1$

$\boxed{y(5) = 5}$

$$y(6) = b_{-2}\, w(8) + b_{-1}\, w(7) + b_0\, w(6) + b_1\, w(5) + b_2\, w(4)$$

$$= \tfrac{1}{3}(0) + \tfrac{2}{3}(0) + 1(6) + \tfrac{2}{3}(0) + \tfrac{1}{3}(0)$$

$$\boxed{y(6) = 6}$$

$$y(7) = b_{-2}\, w(9) + b_{-1}\, w(8) + b_0\, w(7) + b_1\, w(6) + b_2\, w(5)$$

$$= \frac{1}{\cancel{3}}(\cancel{9})^{3} + \tfrac{2}{3}(0) + 1(0) + \frac{2}{\cancel{3}}(\cancel{6})^{2} + \tfrac{1}{3}(0)$$

$$y(7) = 3 + 4$$

$$\boxed{y(7) = 7}$$

$$y(8) = b_{-2}\, w(10) + b_{-1}\, w(9) + b_0\, w(8) + b_1\, w(7) + b_2\, w(6)$$

$$= \tfrac{1}{3}(0) + \frac{2}{\cancel{3}}(\cancel{9})^{3} + 1(0) + \tfrac{2}{3}(0) + \frac{1}{\cancel{3}}(\cancel{6})^{2}$$

$$y(8) = 6 + 2$$

$$\boxed{y(8) = 8}$$

$$y(9) = b_{-2}\, w(11) + b_{-1}\, w(10) + b_0\, w(9) + b_1\, w(8) + b_2\, w(7)$$

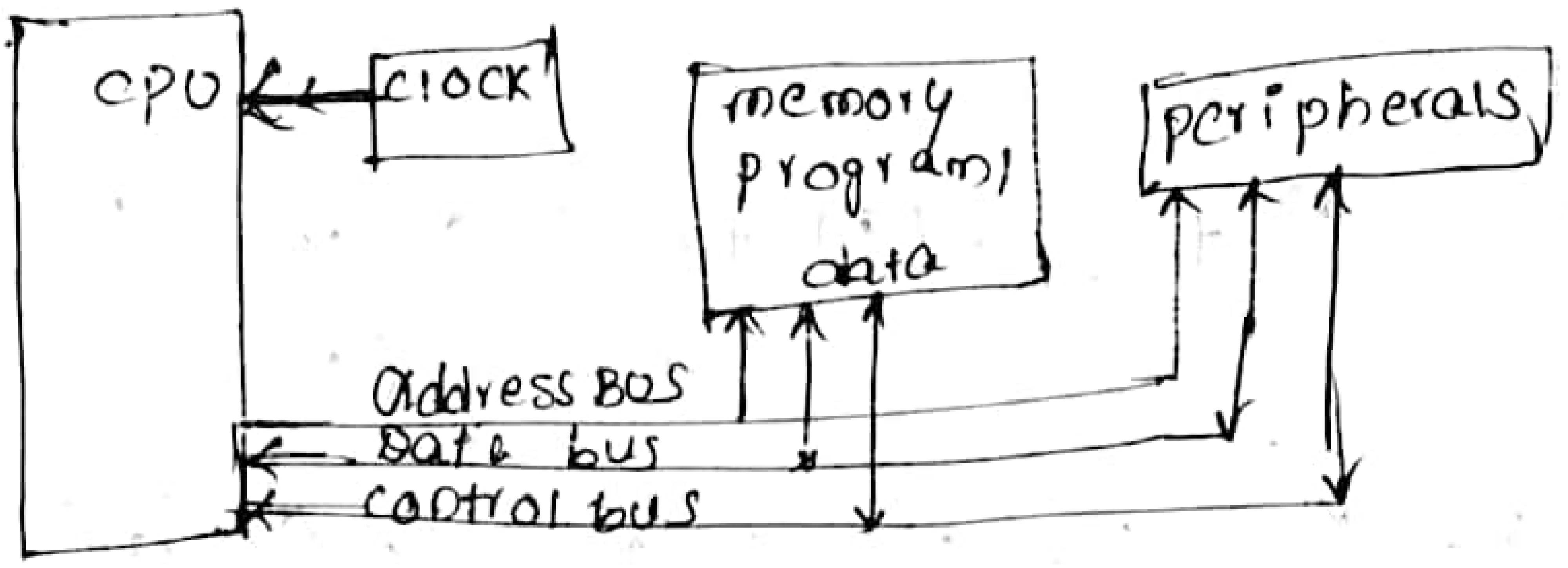
$$= \tfrac{1}{3}(0) + \tfrac{2}{3}(0) + 1(9) + \tfrac{2}{3}(0) + \tfrac{1}{3}(0)$$

$$\boxed{y(9) = 9}$$

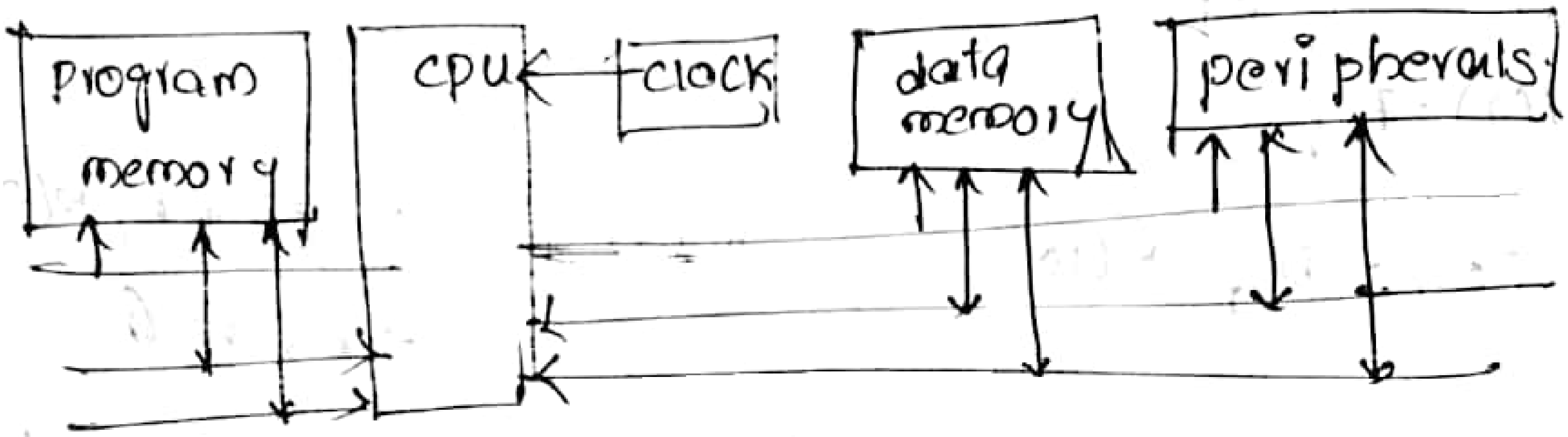
$$y(m) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

⟹ The PDSP's are divided into
1) General purpose 2) special purpose DSP's.

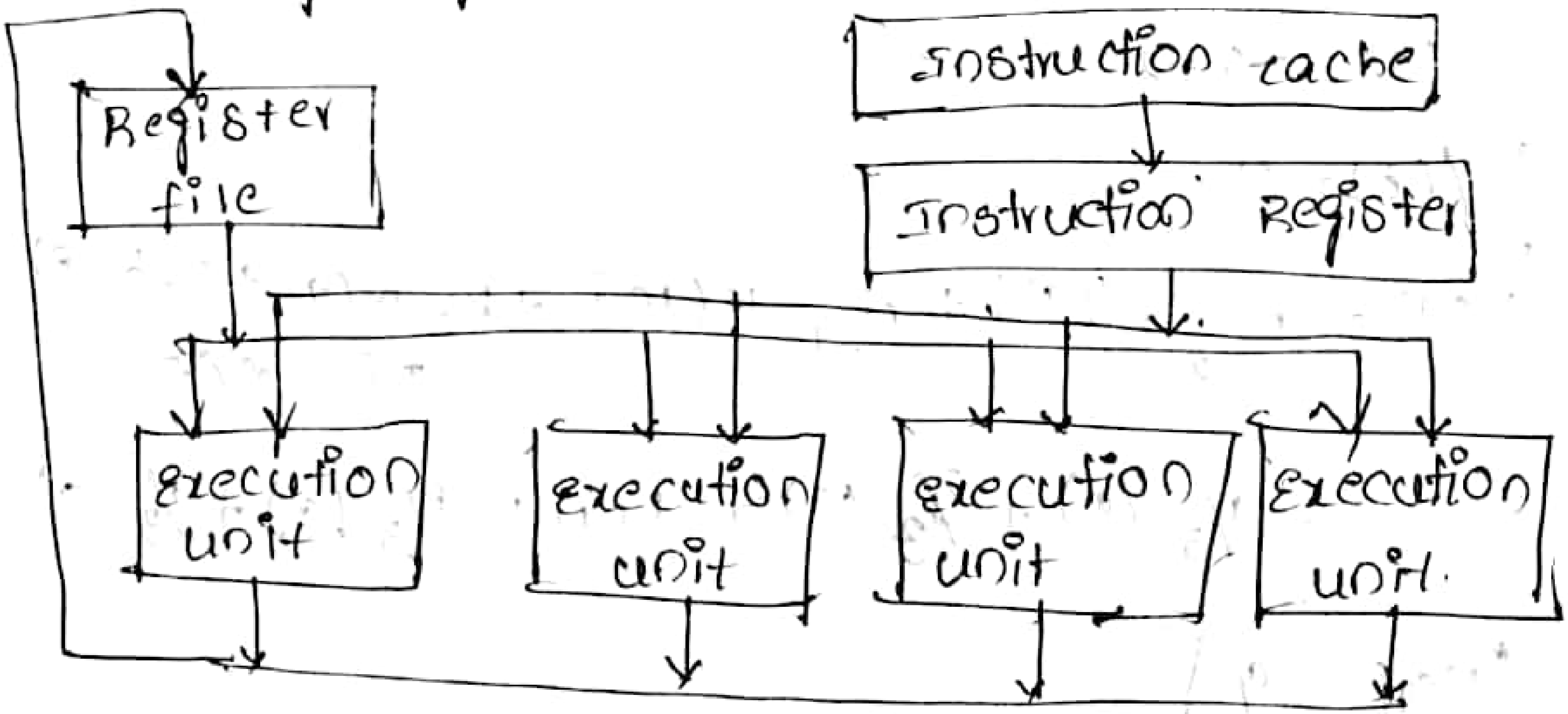
1) Von Neumann Architecture:



2) Harvard Architecture:



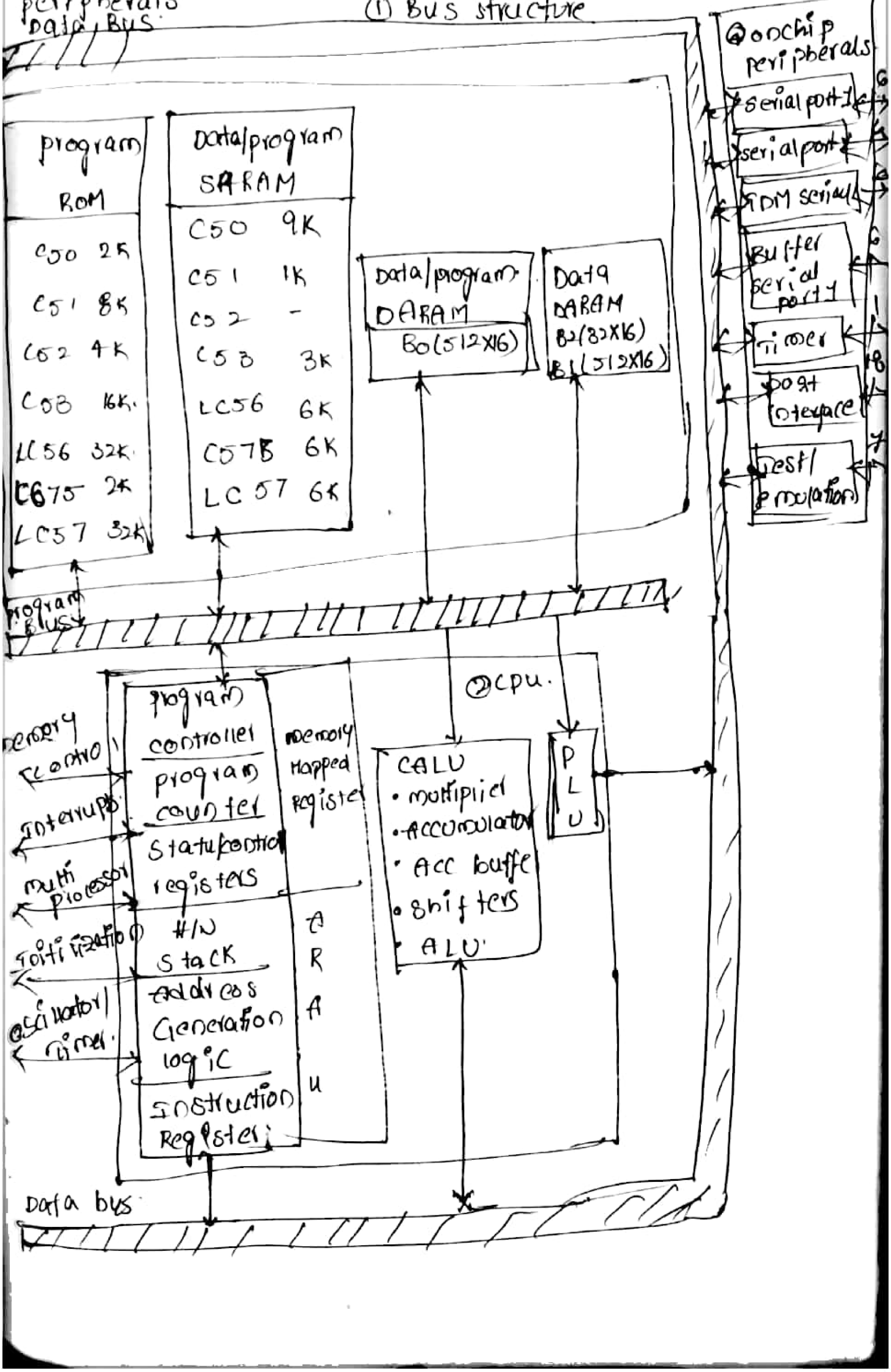
3) VLIW (very long Instruction word) Architecture:



Architecture of TMS320C50:

⇒ It is divided into 4 sub blocks.

1) Bus structure 2) CPU 3) onchip memory 4) on chip peripherals.



Code No: RT32042 **R13** SET - 1

III B. Tech II Semester Supplementary Examinations, October/November - 2020

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

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PART –A (22 Marks)

1 a) Check whether the following system is i) Linear, and ii)Time invariant. $y(n+2)+2y(n)=x(n+1)+2$. [4M]

b) How FFT is more efficient to determine DFT of sequence? [3M]

c) What are the applications of Z-Transforms? [4M]

d) Explain about impulse invariant technique. [4M]

e) Draw the schematic of interpolator. [3M]

f) What are the different stages in pipelining? [4M]

PART –B (48 Marks)

2 a) Determine the frequency response, and time delay of the system given by. $y(n)=x(n)-x(n-1)+x(n-2)$. [8M]

b) Determine whether the following system is: i) Linear ii) Causal iii) Stable, and iv) Time invariant. $y(n)=\log 10|x(n)|$ Justify your answer. [8M]

3 a) Find the DFT of the following sequence using DIF FFT? $x(n)=\{1,2,3,5,5,3,2,1\}$. [8M]

b) Find the inverse FFT of X[k] = [10, -2+j2, 4, -2-j2]. [8M]

4 a) Design an FIR Low Pass filter with $\omega_c = 1.4\pi/s$ *and* $N = 7$ using Hamming window. [8M]

b) Determine the Z-transform of the signals: [8M]
i) $x(n)=nu(n-1)$ ii) $x(n)=2^n\cos(3n)u(n)$.

5 a) What is a Kaiser window? In what way is it superior to other window functions? [8M]

b) Compare and Contrast Butterworth and Chebyshev approximations. [8M]

6 a) Give the time-domain characterization of up – sampler. [8M]

b) Explain about sampling rate conversion [8M]

7 a) Draw and explain the memory architecture of the TMS320C3X processor. [8M]

b) What is bit- reversed addressing mode? Explain. [8M]

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Code No: RT32042 **R13** SET - 1

III B. Tech II Semester Supplementary Examinations, November -2019

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

*****

PART –A (22 Marks)

1\. a) List the properties of DT system. [3M]

b) Find and plot the spectrum of $\delta(n-1)$. [4M]

c) Find the IZT of $X(z) = \frac{z}{z-1}$, for $|z| > 1$ and $|z| < 1$. [4M]

d) Explain the mapping of s-plane to z-plane in impulse invariant transformation. [4M]

e) Give the schematic representation of decimator and interpolator. [4M]

f) What are the important features of programmable digital signal processor? [3M]

PART –B (48 Marks)

2\. a) Discuss the stability of the systems described by the impulse response below: [8M]

i. $h(n) = 2^{-n}u(n)$.

ii. $h(n) = 0.5^n u(n) - 0.5^n u(4-n)$.

b) Determine the steady-state response of the system governed by the following difference equation: $12y(n) - 7y(n-1) + y(n-2) = \sin\left(\frac{\pi}{3}n\right)u(n)$. [8M]

3\. a) Compute the coefficients of the Fourier series of the periodic sequence given below and plot its spectrum. $x(n) = \sin\left(\frac{2\pi n}{N}\right), for\ N = 20$. [8M]

b) Compute the 8-point DFT of the following sequence using radix-2 DITFFT algorithm: $x(n) = \delta(n) + 2\delta(n-1) - \delta(n-2) + \delta(n-3)$. [8M]

4\. a) Compute the time response of the causal system described by the transfer function $H(z) = \frac{(z-1)^2}{z^2-0.32z+0.8}$ when the input signal is the unit step. [8M]

b) Give the direct form-I and direct form-II realizations for the transfer function: [8M]

$$H(z) = 0.0034 + 0.0106z^{-2} + 0.0025z^{-4} + 0.0149z^{-6}.$$

5\. a) Distinguish between FIR and IIR filters. [8M]

b) What are the analog to digital filter transformation techniques? Explain. [8M]

6\. a) What is the difference between single-rate and multi-rate systems? Explain with examples. [8M]

b) Give the frequency domain description of up-sampler. [8M]

7\. Write notes on the following:

a) Specialized addressing modes. [8M]

b) TMS320C5x bus structure. [8M]

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Code No: RT32042 **R13** SET - 1

III B. Tech II Semester Regular/Supplementary Examinations, April - 2017

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Maximum Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

*****

PART –A

1 a) Test whether the following signal is periodic or not ,if periodic find the fundamental period $\sin\sqrt{2}\,\pi\, t$ [4M]

b) Find the DFT of a sequence x(n) ={1 ,1, 2, 2} [4M]

c) Give block diagram representation of linear constant-coefficient difference equations. [4M]

d) By impulse invariant method obtain the digital filter transfer function and the differential equation of the analog filter h(s) =1/s+1 [4M]

e) What are the applications of multi rate DSP? [3M]

f) List special feature of DSP architecture. [3M]

PART -B

2 a) Determine whether each of the following systems defined below is (i) casual (ii) linear (iii) dynamic (iv) time invariant [12M]
(i) $y(n) = \log_{10}[\{x(n)\}]$
(ii) y(n) = x(-n-2)
(iii) y(n) = cosh[nx(n) + x(n-1)]

b) Give the frequency domain representation of discrete time signals. [4M]

3 a) Compute the DFT for the sequence {1, 2, 0, 0, 0, 2, 1, 1}. Using radix -2 DIF FFT and radix -2 DIT- FFT algorithm. [8M]

b) Derive the equation to implement a butterfly structure In DITFFT algorithm. [8M]

4 a) Realize the filter H(z)=$(z^{-1}-a)(z^{-1}-b)\ /\ (1-az^{-1})(1-bz^{-1})$ in cascade and parallel forms. [8M]

b) State and prove time convolution property of Z-Transforms. [8M]

5 a) Obtain the impulse response of digital filter to correspond to an analog filter with impulse response $h_a(t) = 0.5\ e^{-2t}$ and with a sampling rate of 1.0kHz using impulse invariant method. [8M]

b) Compare bilinear transformation and impulse invariant mapping. [8M]

6 a) Explain the decimation and interpolation process with an example. Also find the spectrum. [8M]

b) The sequence x(n)=[0,2,4,6,8] is interpolated using interpolation sequence b_k =[1/2,1,1/2] and the interpolation factor is 2.find the interpolated sequence y(m). [8M]

7 a) Describe the multiplier/adder unit of TMS320c54xx processor with a neat block diagram. [8M]

b) What are interrupts? What are the classes of interrupts available in the TMS320C5xx processor? [8M]

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Code No: RT32042 **R13** SET - 2

III B. Tech II Semester Regular/Supplementary Examinations, April - 2017

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Maximum Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

*****

PART –A

1 a) Test whether the following signal is periodic or not ,if periodic find the fundamental period $\sin20\ \pi t+\sin5\pi t$ [4M]
b) Find the values of WNk, When N=8, k=2 and also for k=3. [4M]
c) Draw the direct form realization of FIR system. [4M]
d) What are the properties of chebyshev filter? [3M]
e) Find the spectrum of exponential signal decimated by factor 2. [4M]
f) What are the advantages of VLIW architecture? [3M]

PART -B

2 a) Determine the impulse response of the filter defined by y(n)=x(n)+by(n-1). 8M]
b) A system has unit sample response h(n) given by h(n)=-1/δ(n+1)+1/2δ(n)-1-1/4 δ(n-1). Is the system BIBO stable? Is the filter causal? Justify your answer. [8M]

3 a) Find the DFT of the sequence x[n]={1,2,3,4,5,6,7,8}. [8M]
b) Explain the use of FFT algorithms in linear filtering and correlation. [8M]

4 a) Determine the cascade and parallel realization for the system transfer function $H(z) = 3(z^2+5z+4) / (2z+1)(z+2)$. [8M]
b) State and prove frequency convolution property of Z-Transforms. [8M]

5 a) Design an ideal high pass filter with a frequency response [8M]
Hd (ejw) = 1 for $\pi/4 \le |w| \le \pi$
= 0 for $|w| \le \pi/4$ Find the values of h(n) for N = 11 using
Hamming window. Find H (z) and determine the magnitude response.
b) Derive the expression for Bi linear Transform. [8M]

6 a) Explain the operation used in DSP to increase the sampling rate. [8M]
b) The sequence x(n)=[0,2,4,6,8] is interpolated using interpolation sequence bk =[1/2,1,1/2] and the interpolation factor is 2.find the interpolated sequence y(m). [8M]

7 a) Explain the different types of interrupts in TMS320C54xx Processors. [8M]
b) Describe any four data addressing modes of TMS320c54xx processor. [8M]

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Code No: RT32042 **R13** SET - 3

III B. Tech II Semester Regular/Supplementary Examinations, April - 2017

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Maximum Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

*****

PART –A

1 a) Test the following systems for time invariance $y(n)=n\,x^2(n)$ [4M]

b) Define DFT and IDFT [4M]

c) What are the applications of Z-Transforms? [4M]

d) What are the advantages of Kaiser widow? [4M]

e) What are "decimation" , “decimation factor “and "down sampling"? [3M]

f) List the on-chip peripherals [3M]

PART -B

2 a) Determine and sketch the magnitude and phase response of the following systems [12M]
(i) y(n) = 1/3 [x(n) + x(n-1) + x(n-2)]
(ii) y(n) = ½[x(n) – x(n-1)] (iii) y(n) - 1/2y(n-1)=x(n)

b) Determine the impulse response of the filter defined by y(n)=x(n)+by(n-1) . [4M]

3 a) Determine IDFT of the following [8M]
(i) X(k)={1,1-j2,-1,1+j2} (ii)X(k)={1,0,1,0}

b) Find the DFT of the sequence x[n]={1,2,3,4,5,6,7,8} using DITFFT. [8M]

4 a) Obtain the direct form I, direct form II and Cascade form realization of the following system functions. [8M]
Y(n) = 0.1 y(n-1) + 0.2 y(n-2) + 3x(n) + 3.6 x(n-1) + 0.6 x(n-2).

b) Explain Transposed forms. [8M]

5 a) Comparison of FIR and IIR filters. [8M]

b) What is Hamming Window function? Obtain its frequency domain characteristics. [8M]

6 a) What is Multi Rate Signal Processing? Explain any two applications of multirate signal processing. [8M]

b) Derive the Frequency domain Transfer function of a Decimator. [8M]

7 a) List the major architectural features used in DSP system to achieve high speed program execution. [8M]

b) With examples explain the different addressing formats supported by DSP processors for various signal processing applications. [8M]

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Code No: RT32042 **R13** SET - 4

III B. Tech II Semester Regular/Supplementary Examinations, April - 2017

DIGITAL SIGNAL PROCESSING

(Electronics and Communication Engineering)

Time: 3 hours Maximum Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is compulsory
3. Answer any **THREE** Questions from **Part-B**

*****

PART –A

1 a) Test the following systems for time invariance a $^{x(n)}$. [3M]

b) What are the advantages of FFT over DFT. [4M]

c) Find the Z-transform of $x(n)= (1/8)^n\ u(n)$ and its ROC. [4M]

d) What is the necessary and sufficient condition for linear phase Characteristics in FIR filter? [4M]

e) Explain the term up sampling and down sampling. [3M]

f) What are the different stages in pipelining? [4M]

PART -B

2 a) A system has unit sample response h(n) given by $h(n)=-1/\delta(n+1)+1/2\delta(n)-1-1/4\ \delta(n-1)$. Is the system BIBO stable? Is the filter causal? Justify your answer [8M]

b) Give the frequency domain representation of discrete time signals and systems. [8M]

3 a) How is the FFT algorithm applied to determine inverse discrete Fourier transform? [8M]

b) Derive the equation to implement a butterfly structure In DIFFFT algorithm [8M]

4 a) Obtain the direct form I, direct form II and Cascade form realization of the following system functions. [8M]

$Y(n) = 0.1\ y(n-1) + 0.2\ y(n-2) + 3x(n) + 3.6\ x(n-1) + 0.6\ x(n-2)$.

b) Prove that FIR filter has linear phase if the unit impulse response satisfies the condition $h(n)=h(N-1-n)$, n=0,1,……M-1. Also discuss symmetric and antisymmetric cases of FIR filter. [8M]

5 a) Determine H(Z) for a Butterworth filter satisfying the following specifications: [8M]

$$0.8 \le |H(e^{j\omega}| \le 1, \text{ for } 0\le \omega\le \pi/4$$

$$|H(e^{j\omega}| \le 0.2, \text{ for } \pi/2\le \omega\le \pi$$

Assume T= 0.1 sec. Apply bilinear transformation method

b) Use bilinear transformation method to obtain H(Z) if T= 1 sec and H(s) is $1/(s+1)(S+2)$, $1/(s2+\sqrt{2}\ s +1)$. [8M]

6 a) With necessary derivation explain the operation of sampling rate conversion by a non integer. [8M]

b) The sequence x(n) = [0,3,6,9] is interpolated using interpolation sequence bk=[1/3, 2/3,1,2/3,1/3] and the interpolation factor of 3. Find the interpolated sequence y (m). [8M]

7 a) Explain Memory Access schemes in DSPs. [8M]

b) Explain the memory interface block diagram for the TMS 320 C5x processor. [8M]

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